**LEARNER GUIDE**

Numeracy Level 2

**Unit Standard 7480 Level 2 Credits 3**

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**PERSONAL INFORMATION**

|  |  |
| --- | --- |
| ***NAME*** |  |
| ***CONTACT ADDRESS*** |  |
|  |
| ***Code*** |  |
| ***Telephone (H)*** |  |
| ***Telephone (W)*** |  |
| ***Cellular*** |  |
| ***Learner Number*** |  |
| ***Identity Number*** |  |
| ***EMPLOYER*** |  |
| ***EMPLOYER CONTACT ADDRESS*** |  |
|  |
| ***Code*** |  |
| ***Supervisor Name*** |  |
| ***Supervisor Contact Address*** |  |
|  |
| ***Code*** |  |
| ***Telephone (H)*** |  |
| ***Telephone (W)*** |  |
| ***Cellular*** |  |

**INTRODUCTION**

***Welcome to the learning programme***

Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have learnt. Take notes and share information with your colleagues. Important and relevant information and skills are transferred by sharing!



This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you!

## Structure

### Programme methodology



The programme methodology includes facilitator presentations, readings, individual activities, group discussions and skill application exercises.

**Know what you want to get out of the programme from the beginning and start applying your new skills immediately. Participate as much as possible so that the learning will be interactive and stimulating.**

The following principles were applied in designing the course:

* Because the course is designed to maximise interactive learning, you are encouraged and required to participate fully during the group exercises
* As a learner you will be presented with numerous problems and will be required to fully apply your mind to finding solutions to problems before being presented with the course presenter’s solutions to the problems
* Through participation and interaction the learners can learn as much from each other as they do from the course presenter
* Although learners attending the course may have varied degrees of experience in the subject matter, the course is designed to ensure that all delegates complete the course with the same level of understanding
* Because reflection forms an important component of adult learning, some learning resources will be followed by a self-assessment which is designed so that the learner will reflect on the material just completed.

This approach to course construction will ensure that learners first apply their minds to finding solutions to problems before the answers are provided, which will then maximise the learning process which is further strengthened by reflecting on the material covered by means of the self-assessments.

***Different role players in delivery process***

* Learner
* Facilitator
* Assessor
* Moderator

### What Learning Material you should have

This learning material has also been designed to provide the learner with a comprehensive reference guide. It is important that you take responsibility for your own learning process; this includes taking care of your learner material. You should at all times have the following material with you:

|  |  |
| --- | --- |
| ***Learner Guide*** | ***This learner guide is your valuable possession:***This is your textbook and reference material, which provides you with all the information you will require to meet the exit level outcomes. During contact sessions, your facilitator will use this guide and will facilitate the learning process. During contact sessions a variety of activities will assist you to gain knowledge and skills. Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have learnt. Take and share information with your colleagues. Important and relevant information and skills are transferred by sharing!This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you! |
| ***Formative Assessment Workbook*** | The Formative Assessment Workbook supports the Learner Guide and assists you in applying what you have learnt. The formative assessment workbook contains classroom activities that you have to complete in the classroom, during contact sessions either in groups or individually.You are required to complete all activities in the Formative Assessment Workbook. The facilitator will assist, lead and coach you through the process. These activities ensure that you understand the content of the material and that you get an opportunity to test your understanding.  |

### Different types of activities you can expect

To accommodate your learning preferences, a variety of different types of activities are included in the formative and summative assessments. They will assist you to achieve the outcomes (correct results) and should guide you through the learning process, making learning a positive and pleasant experience.



The table below provides you with more information related to the types of activities.

| ***Types of Activities*** | ***Description*** | ***Purpose*** |
| --- | --- | --- |
| ***Knowledge Activities*** | You are required to complete these activities on your own.  | These activities normally test your understanding and ability to apply the information. |
| ***Skills Application Activities*** | You need to complete these activities in the workplace  | These activities require you to apply the knowledge and skills gained in the workplace |
| ***Natural Occurring Evidence*** | You need to collect information and samples of documents from the workplace. | These activities ensure you get the opportunity to learn from experts in the industry.Collecting examples demonstrates how to implement knowledge and skills in a practical way |

### Assessments

The only way to establish whether a learner is competent and has accomplished the specific outcomes is through the assessment process. Assessment involves collecting and interpreting evidence about the learners’ ability to perform a task.

**To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.**

**This programme has been aligned to registered unit standards. You will be assessed against the outcomes as stipulated in the unit standard by completing assessments and by compiling a portfolio of evidence that provides proof of your ability to apply the learning to your work situation.**



***How will Assessments commence?***

***Formative Assessments***

The assessment process is easy to follow. You will be guided by the Facilitator. Your responsibility is to complete all the activities in the Formative Assessment Workbook and submit it to your facilitator.

***Summative Assessments***

You will be required to complete a series of summative assessments. The Summative Assessment Guide will assist you in identifying the evidence required for final assessment purposes. You will be required to complete these activities on your own time, using real life projects in your workplace or business environment in preparing evidence for your Portfolio of Evidence. Your Facilitator will provide more details in this regard.

**To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.**

### Learner Support

**The responsibility of learning rests with you, so be proactive and ask questions and seek assistance and help from your facilitator, if required.**



Please remember that this Skills Programme is based on outcomes based education principles which implies the following:

* You are responsible for your own learning – make sure you manage your study, research and workplace time effectively.
* Learning activities are learner driven – make sure you use the Learner Guide and Formative Assessment Workbook in the manner intended, and are familiar with the workplace requirements.
* The Facilitator is there to reasonably assist you during contact, practical and workplace time for this programme – make sure that you have his/her contact details.
* You are responsible for the safekeeping of your completed Formative Assessment Workbook and Workplace Guide
* If you need assistance please contact your facilitator who will gladly assist you.
* If you have any special needs please inform the facilitator

## Learner Administration



***Attendance Register***

You are required to sign the Attendance Register every day you attend training sessions facilitated by a facilitator.

***Programme Evaluation Form***

On completion you will be supplied with a “Learning programme Evaluation Form”. You are required to evaluate your experience in attending the programme.

Please complete the form at the end of the programme, as this will assist us in improving our service and programme material. Your assistance is highly appreciated.

### Learner Expectations

Please prepare the following information. You will then be asked to introduce yourself to the instructor as well as your fellow learners



|  |
| --- |
| Your name:  |
|  |
|  |
| The organisation you represent:  |
|  |
|  |
| Your position in organisation:  |
|  |
|  |
| What do you hope to achieve by attending this course / what are your course expectations? |
|  |
|  |
|  |
|  |
|  |
|  |
|  |
|  |

# UNIT STANDARD 7480

#### Unit Standard Title

Demonstrate understanding of rational and irrational numbers and number systems

#### NQF Level

2

#### Credits

3

#### Purpose

This unit standard will be useful to people who aim to achieve recognition at some level in Further Education and Training or to meet the Fundamental requirement of a wide range of qualifications registered on the National Qualifications Framework

#### Learning Assumptions

The credit value is based on the assumption that people starting to learn towards this unit standard are competent in Mathematics and Communications at NQF level 1.

#### Range

Approximation in relation to the use of computing technologies, the distinction between exact and approximate answers in a variety of problem settings and measurement error in relation to the accuracy of instruments

More detailed range statements are provided for specific outcomes and assessment criteria as needed

#### Specific Outcomes and Assessment Criteria

**Specific Outcome 1**: Use and analyse computational tools and strategies, and make estimates and approximations.

Range: This outcome includes the need to

* use technology such as calculators
* demonstrate understanding of mathematical relationships and principles involved in computations
* find rational approximations to irrational numbers

**Assessment Criteria**

* Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem.
* Algorithms are executed appropriately in calculations.
* Solutions involving irrational numbers are reported or recorded to degrees of accuracy appropriate to the problem.
* Measurements are reported or recorded in accordance with the degree of accuracy of the instrument used.
* Estimates and approximations are used appropriately in terms of the situation and distinctions are made between the appropriate use of estimates versus approximations: Technological and non-technological settings.
* The roles and limitations of particular algorithms are identified in terms of efficiency and the complexity of the algebraic formulation.
* The viability of selected algorithms is verified and justified in terms of appropriateness to context and efficiency.

**Specific Outcome 2**: Demonstrate understanding of numbers and relationships among numbers and number systems. Notes: Demonstrate understanding of numbers and relationships among numbers and number systems, and represent numbers in different ways

Range: This outcome includes the need to:

* work with rational and irrational numbers
* explore repeating decimals and convert them to common fraction form
* use scientific notation for small and large numbers

**Assessment Criteria**

* Notation for expressing numbers is consistent with mathematical conventions.
* Methods of calculation and approximation are appropriate to the problem types.
* Numbers and quantities are represented using rational and irrational numbers as appropriate to the context.
* Scientific notation is used appropriately and consistently with conventions. Situations for the use of scientific notation are provided and described in terms of advantages.
* Conversions between numbers expressed in different ways: Between decimal and scientific notation and between repeating decimals and common fractions.

#### Unit Standard Essential Embedded Knowledge

The following essential embedded knowledge will be assessed through assessment of the specific outcomes in terms of the stipulated assessment criteria. Candidates are unlikely to achieve all the specific outcomes, to the standards described in the assessment criteria, without knowledge of the listed embedded knowledge. This means that the possession or lack of the knowledge can be inferred directly from the quality of the candidate’s performance against the standards

* Number systems and rational and irrational numbers
* Estimation and approximation

#### Critical cross-field outcomes

Upon successful completion of this course, the learner will be able to:

* Collect, analyse, organise and critically evaluate information: Gather, organise, evaluate and interpret numerical information
* Use mathematics: Use mathematics to analyse, describe and represent realistic and abstract situations and to solve problems
* Communicate effectively: Use everyday language and mathematical language to describe relationships, processes and problem solving methods

# Computational Tools

#### Outcome

Use and analyse computational tools and strategies, and make estimates and approximations.

#### Outcome Notes

This outcome includes the need to

* Use technology such as calculators
* Demonstrate understanding of mathematical relationships and principles involved in computations
* Find rational approximations to irrational numbers

#### Assessment criteria

* Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem
* Algorithms are executed appropriately in calculations
* Solutions involving irrational numbers are reported or recorded to degrees of accuracy appropriate to the problem
* Measurements are reported or recorded in accordance with the degree of accuracy of the instrument used
* Estimates and approximations are used appropriately in terms of the situation and distinctions are made between the appropriate use of estimates versus approximations: Technological and non-technological settings
* The roles and limitations of particular algorithms are identified in terms of efficiency and the complexity of the algebraic formulation The viability of selected algorithms is verified and justified in terms of appropriateness to context and efficiency

## Computational Tools

**A sequence of calculations that sets out a series of detailed steps enabling a particular result to be obtained, is called an algorithm. The process of long division, for instance, is an algorithm.**

The value of algorithms has been rediscovered with the development of calculating machines. These machines are used in many forms today, varying from pocket calculators to complicated computers.

For the purpose of this course the simple pocket calculator will be discussed as a computational tool.

The liquid crystal display (LCD) of the calculator is made of a liquid crystal, hermetically sealed between two glass plates and caution must be exercised in handling the calculator.

The following general rules must be kept in mind to ensure trouble free operation:

* Do not place the calculator in a location subject to direct sunlight, especially in a car with its windows closed in a hot climate. High temperatures may damage the calculator.
* Avoid locations subject to rapid temperature changes and excessive moisture or dust.
* Do not drop or bump the calculator.
* Always use a soft dry cloth to clean the calculator – not a cloth moistened with any volatile solvent or water.
* If the calculator uses batteries, do not leave the batteries in it for extended periods if the unit is not to be used. Battery acid leakage may damage the calculator.

A wide variety of calculations can be done, depending on the type of calculator used. Specialised calculators are used for example scientific, statistic or financial purposes.

It is a good idea to study the manual of your specific calculator well to be able to maximize the use of your computational tool



## Algorithms

### Addition and Subtraction

#### Example 1

123 + 456 + 789

Key in: 123 + 456 + 789

Answer: 1368

#### Example 2

100 - 25 - 35

Key in: 100 - 25 - 35

Answer: 40

**Pressing the = key gives the answer to the entered formula.**

### Multiplication and Division

#### Example 3

50 x (-2) ÷ 4

Key in: 50 x 2 ± ÷ 4 =

Answer: -25

**NOTE:** To enter a negative number, press the ± (change Sign) key after numeric entry.

#### Example 4

5 + 2 x 3 - 2 ÷ 0.5

Key in: 5 + 2 x 3 -2 ÷ 1 =

Answer: 7

### Parentheses (Brackets)

The parentheses key is used to cluster together a series of operations if it is necessary to do it first. When brackets are used the calculations in brackets take precedence over any other calculation. Calculations within the innermost set of brackets will be performed first.

#### Example 5

12 + 42 ÷ (8 - 6)

Key in: 12 + 42 ÷ ( 8 - 6 ) =

Answer: 33

#### Example 6

( 3 + 4 ) x ( 3 - 1 )

Key in: ( 3 + 4 ) x ( 3 - 1 ) =

Answer: 24

Important: An error will occur should the brackets be omitted!

## Activity 1 (SO1, AC 1, 2, 7)

## Integers and real numbers

**Integers** are whole numbers (e.g. 1, 45, 77…) and **real numbers** are numbers with a decimal point (e.g. 24.59, 2.09 and 9.1)

Note that a whole number is *real* if it is written with a decimal point. Therefore;

 64 is an integer, but

 64.0 is real

## Whole Numbers, Fractions And Rounding

When we say that each number has a value that depends on its place in the range of numbers, we usually shake our heads and say, ‘I know’. In order to understand what this really means, let’s look at a few examples.

#### Example 7

The number 194 is the same as 1×100 + 9×10 + 4. The ‘4’ is in the unit’s place, the ‘9’ is in the ten’s place and the ‘1’ is in the hundred’s place.

Notice how we say the number when we speak and the repetition of the terms ‘Units, Tens and Hundreds’.

### Whole numbers

Whole numbers are numbers without fractions. A dozen eggs consists of 12 eggs and a gross of eggs (not often heard these days) is 12 dozen or 144 eggs. In South Africa numbers greater than 999 are supposed to appear with spaces between each group of 3 digits also called groups of thousands. For example, the following numbers should be written as 874 349 172. However, you will find that computers and calculators are not very friendly to this manner of using numbers. Therefore, this manual uses the US system of commas to separate the groups and the same number appears as 874,349,172.

### Rounding whole numbers

Rounding the values of whole numbers is simple. To round the previous number to the nearest tens (10), look at the ten’s position of the number (72) and decide if it is closer to 70 or closer to 80. In this case the number is closer to 70 so 874,349,172 rounded to the nearest 10 is 874,349,170.

To round 874,349,172 to the nearest 100, look at the hundred’s position (and all numbers to its right) and get 170. Now decide if 170 is closer to 100 or closer to 200. It’s closer to 200. Therefore the number 874,349,172 rounded to the nearest 100 is 874,349,200.

The table below shows the number and rounds it as shown.

|  |  |  |  |
| --- | --- | --- | --- |
| **Number** | **Round to nearest** | Look at | **Result** |
| 874,349,172 | 10 | 72 | 874,349,170 |
| 874,349,172 | 100 | 172 | 874,349,200 |
| 874,349,172 | 1,000 | 9,172 | 874,349,000 |
| 874,349,172 | 10,000 | 49,172 | 874,350,000 |
| 874,349,172 | 100,000 | 349,172 | 874,300,000 |
| 874,349,172 | 1,000,000 | 4,349,172 | 874,000,000 |
| 874,349,172 | 10,000,000 | 74,349,172 | 870,000,000 |
| 874,349,172 | 100,000,000 | 874,349,172 | 900,000,000 |
| 874,349,172 | 1,000,000,000 | 874,349,172 | 1,000,000,000 |

By convention the number ‘5’ in a specific position rounds up to the nearest 10, 100 and so forth. For example, 35 rounds up to 40 as the nearest 10, 150 rounds up to 200 as the nearest 100 and 2,500 rounds up 3,000 as the nearest 1,000. This rounding technique is called conventional rounding or popular rounding.

### Fractions

Fractions are very similar to whole numbers but they represent a part of a whole and are less than one. In South Africa, fractional parts of a number are supposed to be represented by a comma. However, this is not what computers and calculators like to use.

Therefore, the US system of representing the fraction part of a number with a period (full stop) is used in this manual.

An example of a fraction would be half an apple that may be represented as 0.5 of an apple (or  an apple). It is important to remember that fractions of a whole mean how many pieces there are when the whole is divided into sections. The more pieces you divide something into, the smaller each piece becomes. As you already know, half a pie is larger than a quarter of a pie. Sometimes this concept is difficult to visualize.

The more something is divided, the smaller each piece becomes. At the same time, the more it is divided, the more pieces there are.

**Fractions may be combined with whole numbers.**



### Rounding fractions

The following table lists the various terms used to represent the fraction parts of numbers as well as the terms used to represent the whole parts of numbers. Notice that the fraction parts are very similar to the whole numbers. Instead of starting at the decimal point and moving to the left as whole numbers do, fractions start at the decimal point and move to the right. The names of the fractional parts correspond to the names of the whole numbers but end in ‘th’ instead of ‘s’.

|  |  |  |
| --- | --- | --- |
| **Term** | **Alternate term** | **Value** |
| Millionth | 6 decimals | 0.000001 |
| Hundred-thousandth | 5 decimals | 0.00001 |
| Ten-thousandth | 4 decimals | 0.0001 |
| Thousandth | 3 decimals | 0.001 |
| Hundredth | 2 decimals | 0.01 |
| Tenth | 1 decimal | 0.1 |
| Units | Whole number | 1 |
| Tens |  | 10 |
| Hundreds |  | 100 |
| Thousands |  | 1,000 |
| Ten-thousands |  | 10,000 |
| Hundred-thousands |  | 100,000 |
| Millions |  | 1,000,000 |

*Terms, alternate terms and their values*

#### Example 8

When you next buy your favourite cool drink or milk in a bottle, look at all the bottles and notice how the contents vary among them. I have a tin of fruit juice in front of me that says it contains 340ml. I can’t see it but I now know that 340ml is an approximation to the amount of liquid in the tin. I also know that the difference is acceptable to me or I would have stopped buying this brand of cool drink a long time ago.

Below is the range of several values called ‘Minimum’ and ‘Maximum’ that round to 340ml for various rounding values (‘Round to’).

|  |  |  |  |
| --- | --- | --- | --- |
| Value | **Round to** | **Minimum** | **Maximum** |
| 340ml | 1ml | 339.5ml | 340.4ml |
| 340ml | 10ml | 335ml | 344ml |
| 340ml | 20ml | 330ml | 349ml |

Values that round to 340ml for different precisions

In order to find the range (minimum and maximum values) for any given value and rounding value, following these steps.

1. Call the original value you have ‘Value’.
2. Divide the rounding value in half and call it ‘Range’.
3. Calculate the minimum value as (Value – Range) and call it ‘Minimum’.
4. Subtract one (1) from the last digit of ‘Range’. Note that you must ignore the decimal point when doing this. Call this number ‘Adjusted Range’.
5. Add the ‘Adjusted Range’ to ‘Value’ (Value + Adjusted Range) and call it ‘Maximum’.

If we use the example of the cool drink and we are told that the rounding value is 1ml, this is the same as saying that the last digit is accurate to within 1ml and has been rounded to 340ml. The following steps show how to calculate the range of numbers when you know the value and its accuracy.

1. ‘Value’ = 340ml.
2. ‘Range’ = 0.5ml (1ml ÷ 2 = 0.5ml).
3. ‘Minimum’ = 339.5ml (340ml – 0.5ml = 339.5ml).
4. ‘Adjusted Range’ = 0.4ml (subtract 1 from the last digit of ‘Range’).
5. ‘Maximum’ = 340.4ml (340ml + 0.4ml)

#### Example 9

What numbers round to 1234.56 to the nearest hundredth (0.01)?

1. ‘Value’ = 1234.56.
2. ‘Range’ = 0.005 (0.01 ÷ 2).
3. ‘Minimum’ = 1234.555 (1234.56 – 0.005).
4. ‘Adjusted Range’ = 0.004 (reduce last digit by 1).
5. ‘Maximum’ = 1234.564 (1234.56 + 0.004).

Determining the range of values that round to a value is a bit more work than rounding a number to a specific accuracy.

## Groups Of Numbers

Not all numbers are exact – some are actually infinite. This makes it difficult or impossible to represent them exactly. The main groups of numbers will be discussed below.

All Real numbers, (numbers with real values) can be divided into Rational and Irrational numbers.

Rational numbers have values that can be determined exactly and Irrational numbers have values that cannot be exactly determined. Rational numbers are very often perfect squares.

|  |
| --- |
| Real numbers |
| Rational numbers | Irrational numbers |
| ⅔; √4; √9; √(16/25) | √5; 3√7 |

Rational numbers are all numbers that can be represented as a ratio () of two numbers. All whole numbers are rational numbers because they may be represented as their value over 1. The number 3 is therefore .

Irrational numbers are those that cannot be represented as a ratio of two whole numbers: this means numbers that cannot be represented as simple fractions. Irrational numbers cannot be represented as terminating or repeating decimals

Two irrational numbers are  and π. When dealing with arithmetic involving irrational numbers it is best to keep them as they appear unless an approximate answer is required.

### The effect of error in calculations:

From the explanation it can be seen that not all answers are hundred percent correct even if our arithmetic was correct. It depends on more than that. Often we use values that were measured and these can never be perfectly accurate. This is discussed in more detail in the next section.

## Activity 2: (SO1, AC3 – 6)

# Numbers and Relationships

#### Outcome

Demonstrate an understanding of numbers and relationships among numbers and number systems, and represent numbers in different ways

#### Outcome Notes

This outcome includes the need to

* work with rational and irrational numbers
* explore repeating decimals and convert them to common fraction form
* use scientific notation for small and large numbers

#### Assessment criteria

* Notation for expressing numbers is consistent with mathematical conventions
* Methods of calculation and approximation are appropriate to the problem types.
* Numbers and quantities are represented using rational and irrational numbers as appropriate to the context
* Scientific notation is used appropriately and consistently with conventions. Situations for the use of scientific notation are provided and described in terms of advantages
* Conversions between numbers expressed in different ways are accurate: Between decimal and scientific notation and between repeating decimals and common fractions

## Decimal Number System

There is more than one number system. The decimal number system is the most common where there are 10 elements

#### Elements:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

#### Base number:

10

The decimal system is the one we use to count and perform operations on numbers. Perhaps the decimal system is used because humans have ten fingers and counting objects began with using our ten digits, as they are called. I wonder how we would be counting if we had eight or twelve fingers. I suppose we will never know.

When the decimal system is mentioned, almost everyone says that they know and understand the system. However, when asked to count, they start at 1 and count to 10. This is incorrect! The decimal system consists of ten items but they actually start at 0 and end at 9.

The Arabic nations invented a method to represent two extremely important concepts:

* how to represent zero and
* a system so that each number in a specific position has a well-known and easily definable meaning or weight.

It may sound strange but the invention of zero (or nothing) was the biggest breakthrough in mathematical history. together with the fact that the position of each digit had a specific meaning meant that commerce and trade were simplified. The western world uses slight modifications of the numbering figures invented so long ago by the Arabic nations.

### Hierarchy of Decimal Numbers

|  |  |  |
| --- | --- | --- |
| **Number** | **Name** | **How many** |
| 0  | zero |   |
| 1  | one | One |
| 2  | two | two ones |
| 3  | three | Three ones |
| 4  | four | Four ones |
| 5  | five | Five ones |
| 6  | six | Six ones |
| 7  | seven | seven ones |
| 8  | eight | Eight ones |
| 9  | nine | Nine ones |
| 10  | ten | one ten |
| 20 | twenty | two tens |
| 30 | thirty | three tens |
| 40 | forty | four tens |
| 50  | fifty | five tens |
| 60 | sixty | six tens |
| 70 | seventy | seven tens |
| 80 | eighty | eight tens |
| 90 | ninety | nine tens |

|  |  |  |
| --- | --- | --- |
| **Number** | **Name** | **How Many** |
| 100  | one hundred | ten tens |
| 1,000  | one thousand | ten hundreds |
| 10,000 | ten thousand | ten thousands |
| 100,000 | one hundred thousand | one hundred thousands |
| 1,000,000 | one million | one thousand thousands |

Some people use a comma to mark every 3 digits. It just keeps track of the digits and makes the numbers easier to read.

### Fractions

Digits to the right of the decimal point represent the fractional part of the decimal number. Each place value has a value that is one tenth the value to the immediate left of it.

|  |  |  |
| --- | --- | --- |
| Number | Name | Fraction |
| .1 | tenth | 1/10 |
| .01 | hundredth | 1/100 |
| .001 | thousandth | 1/1000 |
| .0001 | ten thousandth | 1/10000 |
| .00001 | hundred thousandth | 1/100000 |

#### Example 10

0.234 = 234/1000 (said - point 2 3 4, or 234 thousandths, or two hundred thirty four thousandths)

4.83 = 4 83/100 (said - 4 point 8 3, or 4 and 83 hundredths)

### Repeating decimals

Repeating decimals are decimals with certain digits that repeat.

#### Example 11

 1.3333333 never ends and is written as:

 

However this is a rather complicated method to use. It is much easier to simplify the repeating number to its nearest decimal value. We do however indicate that it is repetitive.

 

Here are a few examples:



Often when we calculate a number we end up with a lot of decimals, this isn’t practical and we round off to a certain number of decimal numbers.

**If a number is between 1 - 4 it is rounded down and if it is between 5-9 it is rounded up.**

### Convert repeating decimals to common fraction form

**Important Note:** any span of numbers that is underlined signifies that those numbers are repeated. For example, 0.09 signifies 0.090909.

**Only fractions in lowest terms are listed.** For instance, to find 2/8, first simplify it to 1/4 then search for it in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| **fraction = decimal** |   |   |   |
| **1/1** = 1 |   |   |   |
| **1/2** = 0.5 |   |   |   |
| **1/3** = 0.3 | 2/3 = 0.6 |   |   |
| **1/4** = 0.25 | 3/4 = 0.75 |   |   |
| **1/5** = 0.2 | 2/5 = 0.4 | 3/5 = 0.6 | 4/5 = 0.8 |
| **1/6** = 0.16 | 5/6 = 0.83 |   |   |
| **1/7** =  0.142857 | 2/7 =  0.285714 | 3/7 =  0.428571 | 4/7 =  0.571428 |
|   | 5/7 =  0.714285 | 6/7 =  0.857142 |   |
| **1/8** = 0.125 | 3/8 = 0.375 | 5/8 = 0.625 | 7/8 = 0.875 |
| **1/9** = 0.1 | 2/9 = 0.2 | 4/9 = 0.4 | 5/9 = 0.5 |
|   | 7/9 = 0.7 | 8/9 = 0.8 |   |
| **1/10** = 0.1 | 3/10 = 0.3 | 7/10 = 0.7 | 9/10 = 0.9 |
| **1/11** = 0.09 | 2/11 = 0.18 | 3/11 = 0.27 | 4/11 = 0.36 |
|   | 5/11 = 0.45 | 6/11 = 0.54 | 7/11 = 0.63 |
|   | 8/11 = 0.72 | 9/11 = 0.81 | 10/11 = 0.90 |
| **1/12** = 0.083 | 5/12 = 0.416 | 7/12 = 0.583 | 11/12 = 0.916 |
| **1/16** = 0.0625 | 3/16 = 0.1875  | 5/16 = 0.3125 | 7/16 = 0.4375 |
|   | 11/16 = 0.6875 | 13/16 = 0.8125 | 15/16 = 0.9375 |
| **1/32** = 0.03125 | 3/32 = 0.09375 | 5/32 = 0.15625 | 7/32 = 0.21875 |
|   | 9/32 = 0.28125 | 11/32 = 0.34375 | 13/32 = 0.40625 |
|   | 15/32 = 0.46875 | 17/32 = 0.53125 | 19/32 = 0.59375 |
|   | 21/32 = 0.65625 | 23/32 = 0.71875 | 25/32 = 0.78125 |
|   | 27/32 = 0.84375 | 29/32 = 0.90625 | 31/32 = 0.96875 |

## The Effect Of Error In Calculations

From the explanation it can be seen that not all answers are hundred percent correct even if our arithmetic is correct. It depends on more than that. Often we use values that were measured and these can never be perfectly accurate.

### Significant figures

#### Significant figures

Significant figures help us to work as accurately as possible when working with measured data. A 2 figure measurement, like 91 m is accurate to ±1 m. The percentage error is therefore:

1/91 x 100 = 1, 09% ≈1%

A 4 figure measurement like 91,11m is accurate to 0,01m. The percentage error is therefore:

0, 01/91.11 x 100 = 0, 0109 ≈0, 01

**This is 100 times more accurate!**

When trying to determine the number of significant figures a number has you should not consider zeros before or after the number. Unless the number has a full stop at the end, this is common in American notation.

#### Example: 12

10 has 1 significant figure

0,00234 has 3 significant figures

2340. has 4 significant figures

3.45 has 3 significant figures

#### Rules used to determine the number of significant figures the answer of a calculation should have:

**Multiplication and subtraction:**

Number with the least significant figures determines the answer:

12,345 x 6, 7 = 83 NOT 82, 7115

**Addition and subtraction**

Retain the smallest number of decimal places.

10,345 + 9, 9 = 20.2 NOT 20,245 or 20!

Whole numbers, or integers, suffice for discussing numbers of people or cattle, but entities land, wine and grain often need to be measured out in varying quantities that does not correspond to whole numbers. Among the followers of Pythagoras it was discovered that whole numbers and fractions (known as rational numbers because they can be expressed as ratios) do not account for all numbers.

The numbers that can not be expressed as ratios are called irrational. An example is the square root of 2 (the number that multiplied by itself equals 2). Rational and irrational numbers together represent all numbers greater than zero.

It was not until the Renaissance that the progress of mathematics called for a further extension of the numbers below zero. It was gradually realised that these ”negative” numbers were an acceptable mathematical idea, provided that they are handled consistently.

The concept of negative numbers occurs in every day life. A person can not find a negative number of money in his pocket, but his overdraft at the bank can be negative.

## Estimating The Correct Answer

In certain circumstances it is sufficient to make use of approximations. That is an estimate of a given quantity to a certain degree of accuracy. If, for example, the distance between two points is measured as 2 385 m, then the measurement is correct to the nearest meter.

If, however, this amount of accuracy is not necessary, the distance could be written as 2 384 m – an approximation accurate to the nearest 10m. Acceptable approximation would be determined by the circumstances – if very small quantities are used, approximation to the nearest meter would not be acceptable.

This subsection takes errors produced by rounding and demonstrates how this can be very useful in our daily lives. When we intentionally introduce errors in arithmetic caused by too much rounding, we obtain a ‘ballpark’ estimate of the correct answer. This estimate may be used when shopping to obtain an idea of the total amount our purchases will be as well as what change we should expect when we pay at the till.

We may estimate the answers to calculations by rounding all the numbers sensibly. Often the rounding may be to one significant figure or two at the most. We are really using our knowledge of rounding to check our arithmetic. For example, 33.78 ÷ 17.24 is approximately 34 ÷ 17, which is 2. This is a sensible rounding and we know that the correct answer should be approximately 2.

#### Example 13

A box of imported chocolates cost R27.69. Approximately how much will four boxes cost? R27.69 rounded to the nearest ten rand is R30 so four boxes would cost approximately R120. If you are told at the till that you owe R194.62, you know there’s a problem!

You may want your approximation a bit more accurate: Round R27.69 to R28.

Note that 28 = 25 + 3: Use parenthesis and multiple the values inside like this: (25 + 3) × 4 = 100 + 12 = R112. With a little practice you can do this all in your head while still holding a conversation.

#### Example 14

Hassim used his calculator to work out 8.623 × 4.710 and wrote his answer down as 406.1433.

Estimate the answer as 9 × 5 = 45. It appears he put the decimal point in the wrong place.

#### Example 15

Thabo is in a hurry to get to work but must carry out the following operations before leaving. The answers are written down rounding to 3 decimal places.

* 3.62 × 8.94 = 32.363
* 47.92 ÷ 2.17 = 1.512
* 184 × 3.616 = 665.344
* (21.4 + 19.7) × 3.61 = 14.837.

Using estimates to check the work we obtain the following:

* Estimate 4 × 9 = 36; the answer could be correct.
* Estimate 50 ÷ 2 = 25; the answer must be incorrect.
* Estimate 200 × 4 = 800; the answer could be correct.
* Estimate (20 + 20) × 4 = 160; the answer must be incorrect.

Rounding used with arithmetic is a shortcut to obtain an approximate answer. In order to ensure that you obtain the correct answer when performing calculations:

* Estimate the answer mentally.
* Calculate the answer (with or without a calculator).
* Check that the calculated answer is sensible by comparing it to your mental estimate.

#### Example 16

On the next page is a picture of a man standing next to a large building. Estimate the height of the building.

The building appears to be about five times the height of the man standing near the building. Assuming that the man is 1.8m tall, the building is about 9m tall.



## Scientific Notation

Scientific notation is very useful as it allows you to express very large or small numbers easily and is also helpful in preventing ambiguity in regard to significant figures. Imagine you have to travel to Dar-Es-Salaam by bicycle and you have a sponsor that is willing to pay you per metre travelled – scientific notation will help you to express this very large amount more easily. Winning 35 million on the Lotto is another example.

Very small amounts: scientists, chemists, etc at times work in minute amounts and scientific notation gives them a short way of expressing these small numbers.

For example: 1000 could be construed as having 1 significant figure or 4, however if it is indicated as 1 x 103  we know it only has 1 or 1,000 x 103 and we know it has 4.

#### Example 23

* 1 km = 1 x 1000 m = 1 x 103m
* 12345m=1,2345 x 104m
* 0,000012m = 1,2 x 10-5m
* 0,12m = 1,2 x 10-1m
* 1mm = 1/1000m = 1 x 10-3m

### Using scientific notation

Calculate (1,234 x 106) + (6, 7 x 103)

Solution:

(1,234 x 106) + (6, 7 x 103)

= (1,234 x 106) + (0, 0067 x 106)

= (1,234 + 0, 0067) x 106

= 1, 2407 x 106

≈ 1,241 x 106