LEARNER GUIDE

Numeracy Level 3

Unit Standard 9010 Level 3 Credits 2

Unit Standard 9013 Level 3 Credits 4

Unit Standard 9012 Level 3 Credits 5

Unit Standard 7456 Level 3 Credits 5

TABLE OF CONTENTS

DOCUMENT CHECKLIST

Document	✓/✗
Strategy and alignment Matrix	
Learner Guides	
Learner Formative Assessment Workbooks	
Summative assessment guide	
Facilitator Guide	
Attendance Register	
Name Tags	
Hand-outs	
Course Evaluation Forms for learners	
Course/Venue Evaluation Forms for facilitator	
Additional Material	
Other:	

Acknowledgement of receipt and preparations	
Facilitators Name and Surname	
Facilitators Signature	
Date	

YOUR ROLE AS FACILITATOR

You, as facilitator, need to ensure that learners have a thorough understanding of the topic presented. Learners must be able to further learning independently and apply their knowledge and skill in the workplace once they have completed the program.

In order to achieve this, you need to have:

1. ***Knowledge of the subject/topic***
   - ✓ Understand the requirements of the unit standard
   - ✓ Be a Subject Matter Expert (SME) or know more than just the basics about the topic
   - ✓ Be able to give examples of how to apply the content in the workplace
2. ***Knowledge of the workplace***
   - ✓ Be able to indicate how learners can/should apply their knowledge and skill in the workplace
3. ***Knowledge of the course content***
   - ✓ Be familiar with the material, layout and content of the course.
   - ✓ Understand the linkage between the different guides and how they should be used.
4. ***Knowledge and understanding of the methodology***
   - ✓ Familiarise yourself with the content of the assessment guides and documents for this unit standard.
   - ✓ Follow the assessment procedure correctly.
   - ✓ Familiarise yourself with the preferred methodology that should be used.
   - ✓ Prepare yourself accordingly.

Training Venue And Training Aid Compliance

Ensure that the following is in place:

Facilitator is in possession of:

- ✓ Facilitator Guide,
- ✓ Learner Guide,
- ✓ Assessment Guide
- ✓ Attendance Register for each training day
- ✓ Handouts, if applicable

Learners are in possession of:

- ✓ Learner Guides
- ✓ Assessment Guides

Training Venue contains:

- ✓ Dictionaries, if available
- ✓ A chair for each learner
- ✓ A desk with sufficient seating space for each learner
- ✓ Paper, pens and pencils for all learners
- ✓ A flip chart stand with flip chart sheets
- ✓ A PC for the slide shows
- ✓ Prestik
- ✓ Whiteboard and permanent markers (3 different colours each)
- ✓ Sufficient ventilation or air-conditioning
- ✓ Schedule of training and relaxation times
- ✓ Name tags for each learner
- ✓ List of classroom conduct, including:

  Smoke breaks

Body breaks

Cell phone etiquette

General classroom etiquette

The Facilitator Guide

It guides you through the duration of the Learning Programme;

As a training aid it contains activities that will assist you in transferring the knowledge and skills as stipulated in the SAQA required specific outcomes and assessment criteria

How to use your facilitator guide

The Facilitator Guide contains the essential information to cover the outcomes as stipulated for this Unit Standard. As such, the content of the Learner Guide has to be covered completely. .

This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority.

These descriptions will define what the learners have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment.

Programme introduction

The following introductory actions are to be taken upon commencement of the programme:

- ✓ Facilitator him/herself to the learners
- ✓ Let the delegates introduce themselves individually.
- ✓ Give a brief comment after each introduction and welcome the delegate before moving to then next delegate. Give full attention to each delegate as they introduce themselves.
- ✓ Let the participants know what time the breaks are, and for how long. Don't forget lunch time. Tell them where the toilets are and what time you intend to finish.
- ✓ Ensure all administrative tasks such as completion of attendance registers, learner detail forms and the like are completed before the programme commences.

Transition

- ✓ Begin with an appropriate activity, such as an icebreaker, a story or a statement.

- ✓ Ask learners to state their expectations of the course and write them down on a flipchart/whiteboard and put up where visible for duration of course.

State course outcomes

- ✓ Go through the objectives and the content before you proceed with your programme.
- ✓ This learning programme forms a part of the National Certificate Business Administration Services Level 4 and introduces a holistic approach to written communication as an introduction to the Secretarial qualification.

Group and individual activities

A certain level of collective general knowledge can be assumed in any group. Encourage lively discussion in class during group activities. Discussions can take the form of brainstorming if necessary.

Ensure that shy and quiet learners are also drawn into the discussion, as learners will have to learn to start thinking for themselves and taking control of the learning process in order to achieve all the outcomes.

All the conclusions made during group activities have to be noted down by learners in order to achieve all the outcomes. Their notes will form part of formative and summative assessments

Teamwork is important in all the unit standards. Divide the learners into groups and make it clear to them that they have to stay in these groups for the duration of the qualification. They have to learn to cope with conflict and with each other even if they do not get along together, since they will not have any say as to who they will be working with when they find themselves in a working environment.

Timelines

SAQA US ID	***UNIT STANDARD TITLE***		
9010	Demonstrate an understanding of the use of different number bases and measurement units and an awareness of error in the context of relevant calculations		
Institutional Learning Duration	Structured Workplace Learning Duration	NQF LEVEL	CREDITS
6 Hours	24 Hours	Level 3	2
SAQA US ID	***UNIT STANDARD TITLE***		

9013	Describe, apply, analyse and calculate shape and motion in 2-and 3-dimensional space in different contexts		
Institutional Learning Duration	Structured Workplace Learning Duration	NQF LEVEL	CREDITS
12 Hours	28 Hours	Level 3	4
SAQA US ID	***UNIT STANDARD TITLE***		
9012	Investigate life and work related problems using data and probabilities		
Institutional Learning Duration	Structured Workplace Learning Duration	NQF LEVEL	CREDITS
15 Hours	35 Hours	Level 3	5
SAQA US ID	***UNIT STANDARD TITLE***		
7456	Use mathematics to investigate and monitor the financial aspects of personal, business and national issues		
Institutional Learning Duration	Structured Workplace Learning Duration	NQF LEVEL	CREDITS
15 Hours	35 Hours	Level 3	5

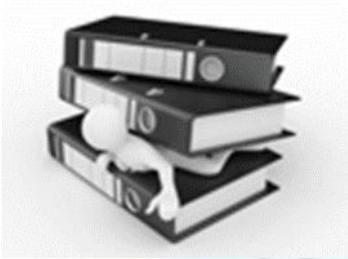
Course content

The course content covers all the basics required for learners to achieve the outcomes, however, at times there is information that learners will have to obtain by themselves.

To this end, a glossary is also not included in the course material. It is expected of learners to compile their own glossaries, which will form part of outcomes for communication unit standards. Please encourage them to start compiling glossaries as soon as possible. The glossaries can be simple: they need only quote the word and the meaning, as long as they understand what it means.

Learners are required to start thinking for themselves and take control of the learning process as soon as possible. To this end, research projects will form part of some assessments.

Class Activities



During and after the initial training the learner will be required to complete a number of class activities. These activities will be both individual and group activities. The activities are numbered and are to be included in the learner's portfolio of evidence. These activities will measure the progress of the learner through the programme. For authenticity reasons these activities must be handwritten, unless indicated otherwise.

Assessment

The process of assessment of competency should be explained in detail to learners upon commencement of course. The following statements should be made:

Attending the training is not sufficient evidence of competence to award a certificate and the credits attached to this programme. Learners are required to undergo assessment in order to prove competence in order to be awarded the credits attached to this programme, eventually leading to a national qualification.

Explain the concept 'Competence'

Competence is the ability to perform whole work roles, to the standards expected in employment, in a real working environment.

There are three levels of competence:

- ✓ ***Foundational competence***: an understanding of what you do and why
- ✓ ***Practical competence***: the ability to perform a set of tasks in an authentic context
- ✓ ***Reflexive competence***: the ability to adapt to changed circumstances appropriately and responsibly, and to explain the reason behind the action

Generally speaking, in the past, education and training institutions became accustomed to awarding certificates based on the amount of time spent in a classroom or training room. In addition, learners were assessed by means of an examination that tested memory as opposed to actual competence. This meant that many learners were awarded certificates and even whole qualifications without ever having to demonstrate that they were able to practically apply their knowledge and skills.

Now, based on the principles of the National Qualifications Framework, in order for the learners to receive a certificate of competence and be awarded credits, they are required to provide evidence of their competence by compiling a portfolio of evidence, which will be assessed by a Stanford Business College assessor.

Explain the concept 'Portfolio of Evidence'

A portfolio of evidence is a structured collection of evidence that proves the learner's efforts, progress and achievement in a specific learning area, and demonstrates competence.

Explain the assessment process

Assessment of competence is a process of making judgments about an individual's competence through matching evidence collected to the appropriate national standards. That is why the evidence in the learner's portfolio should be closely linked to the outcomes and assessment criteria of the unit standards against which the learners are being assessed.

Ideally, formative assessments should minimise the need for re-assessment as the assessor and the candidate will agree to a summative assessment only when they both feel the candidate is ready.

However, candidates who are deemed not yet competent on a summative assessment will be allowed to be re-assessed no more than two times.

When learners have to undergo re-assessment, the following conditions will apply:

- ✓ Specific feedback will be given so that candidates can concentrate on only those areas in which they were assessed as not yet competent
- ✓ Re-assessment will take place in the same situation or context and under the same conditions as the original assessment
- ✓ Only the specific outcomes that were not achieved will be re-assessed

Candidates who are repeatedly unsuccessful will be given guidance on other possible and more suitable learning avenues.

In order for the assessor to assess competence, the portfolio should provide evidence of both knowledge and skills, and of how knowledge and skills were applied in a variety of contexts.

This Candidate's Assessment Portfolio directs the learner in the activities that need to be completed so that competence can be assessed and the credits attached to the programme be awarded.

- ✓ The assessment requirements for this programme are stated at the beginning of each section.

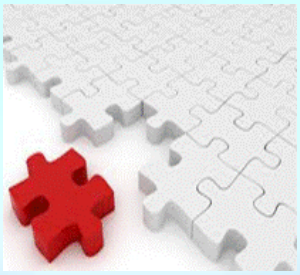
The right of the candidate to appeal against assessment decisions or practices they regard as unfair. An Appeals and Disputes procedure is in place and communicated to all assessment candidates prior to assessment in order for them to appeal on the basis of:

- ✓ Unfair assessment
- ✓ Invalid assessment
- ✓ Unreliable assessment
- ✓ Unethical practices
- ✓ Inadequate expertise and experience of the assessor

Appeals have to be lodged in writing (Candidate Appeal Form) & submitted to the internal moderator within 48 hours, following the assessment in question. The moderator will consider the appeal & make a decision regarding the granting of a re-assessment. The learner will be informed about the appeal-outcome within 3 days of lodging the appeal. Should the learner not be satisfied with the internal appeal outcome, the learner will be advised of the rights to refer the matter to the Services SETA ETQA.

All the required forms and explanations are included in the assessment guide.

Upon Completion Of The Programme



The facilitator should:

- ✓ Ensure that all learners have completed all Learner Administration documentation, including the Learner Evaluation (of the Facilitator) forms
- ✓ All attendance registers have been completed correctly
- ✓ All Learners are in possession of a:

  Learner Guide

  Assessment Guide

  Clear Instructions as to the subsequent activities leading to assessment, moderation and certification

PERSONAL INFORMATION

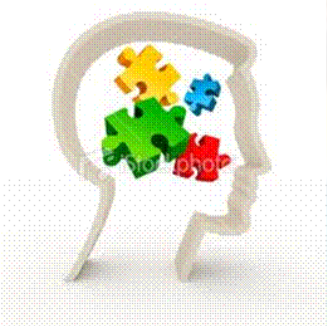
NAME	
CONTACT ADDRESS	
Code	
Telephone (H)	
Telephone (W)	
Cellular	
Learner Number	
Identity Number	

EMPLOYER	
EMPLOYER CONTACT ADDRESS	
Code	
Supervisor Name	
Supervisor Contact Address	
Code	
Telephone (H)	
Telephone (W)	
Cellular	

INTRODUCTION

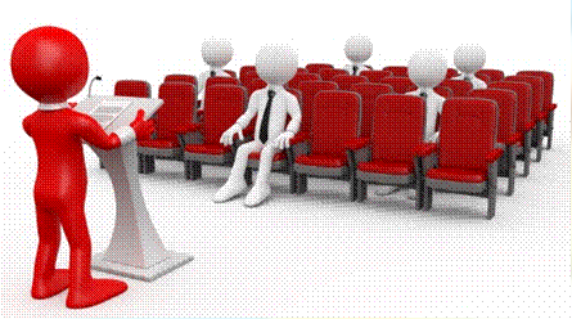
Welcome to the learning programme

Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have learnt. Take notes and share information with your colleagues. Important and relevant information and skills are transferred by sharing!



This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you!

Programme methodology



The programme methodology includes facilitator presentations, readings, individual activities, group discussions and skill application exercises.

Know what you want to get out of the programme from the beginning and start applying your new skills immediately. Participate as much as possible so that the learning will be interactive and stimulating.

The following principles were applied in designing the course:

- ✓ Because the course is designed to maximise interactive learning, you are encouraged and required to participate fully during the group exercises
- ✓ As a learner you will be presented with numerous problems and will be required to fully apply your mind to finding solutions to problems before being presented with the course presenter's solutions to the problems
- ✓ Through participation and interaction the learners can learn as much from each other as they do from the course presenter
- ✓ Although learners attending the course may have varied degrees of experience in the subject matter, the course is designed to ensure that all delegates complete the course with the same level of understanding
- ✓ Because reflection forms an important component of adult learning, some learning resources will be followed by a self-assessment which is designed so that the learner will reflect on the material just completed.

This approach to course construction will ensure that learners first apply their minds to finding solutions to problems before the answers are provided, which will then maximise the learning process which is further strengthened by reflecting on the material covered by means of the self-assessments.

Different role players in delivery process

- ✓ Learner
- ✓ Facilitator
- ✓ Assessor
- ✓ Moderator

What Learning Material you should have

This learning material has also been designed to provide the learner with a comprehensive reference guide.

It is important that you take responsibility for your own learning process; this includes taking care of your learner material. You should at all times have the following material with you:

Learner Guide 	***This learner guide is your valuable possession:*** This is your textbook and reference material, which provides you with all the information you will require to meet the exit level outcomes. During contact sessions, your facilitator will use this guide and will facilitate the learning process. During contact sessions a variety of activities will assist you to gain knowledge and skills. Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have

	learnt. Take and share information with your colleagues. Important and relevant information and skills are transferred by sharing! This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you!
Formative Assessment Workbook 	The Formative Assessment Workbook supports the Learner Guide and assists you in applying what you have learnt. The formative assessment workbook contains classroom activities that you have to complete in the classroom, during contact sessions either in groups or individually. You are required to complete all activities in the Formative Assessment Workbook. The facilitator will assist, lead and coach you through the process. These activities ensure that you understand the content of the material and that you get an opportunity to test your understanding.

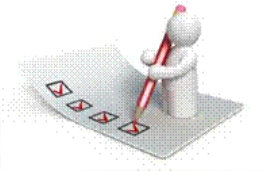
Different types of activities you can expect

To accommodate your learning preferences, a variety of different types of activities are included in the formative and summative assessments. They will assist you to achieve the outcomes (correct results) and should guide you through the learning process, making learning a positive and pleasant experience.

The table below provides you with more information related to the types of activities.

Types of Activities	***Description***	***Purpose***
Knowledge Activities 	You are required to complete these activities on your own.	These activities normally test your understanding and ability to apply the information.
Skills Application Activities 	You need to complete these activities in the workplace	These activities require you to apply the knowledge and skills gained in the workplace
Natural Occurring Evidence 	You need to collect information and samples of documents from the workplace.	These activities ensure you get the opportunity to learn from experts in the industry. Collecting examples demonstrates how to implement knowledge and skills in a practical way

Learner Administration



Attendance Register

You are required to sign the Attendance Register every day you attend training sessions facilitated by a facilitator.

Programme Evaluation Form

On completion you will be supplied with a "Learning programme Evaluation Form". You are required to evaluate your experience in attending the programme.

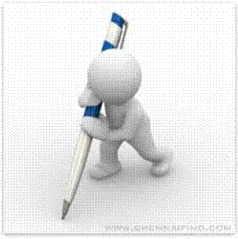
Please complete the form at the end of the programme, as this will assist us in improving our service and programme material. Your assistance is highly appreciated.

Assessments

The only way to establish whether a learner is competent and has accomplished the specific outcomes is through the assessment process. Assessment involves collecting and interpreting evidence about the learners' ability to perform a task.

To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.

This programme has been aligned to registered unit standards. You will be assessed against the outcomes as stipulated in the unit standard by completing assessments and by compiling a portfolio of evidence that provides proof of your ability to apply the learning to your work situation.



How will Assessments commence?

Formative Assessments

The assessment process is easy to follow. You will be guided by the Facilitator. Your responsibility is to complete all the activities in the Formative Assessment Workbook and submit it to your facilitator.

Summative Assessments

You will be required to complete a series of summative assessments. The Summative Assessment Guide will assist you in identifying the evidence required for final assessment purposes. You will be required to complete these activities on your own time, using real life projects in your workplace or business environment in preparing evidence for your Portfolio of Evidence. Your Facilitator will provide more details in this regard.

To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.

Learner Support

The responsibility of learning rests with you, so be proactive and ask questions and seek assistance and help from your facilitator, if required.

Please remember that this Skills Programme is based on outcomes based education principles which implies the following:

- ✓ You are responsible for your own learning – make sure you manage your study, research and workplace time effectively.
- ✓ Learning activities are learner driven – make sure you use the Learner Guide and Formative Assessment Workbook in the manner intended, and are familiar with the workplace requirements.
- ✓ The Facilitator is there to reasonably assist you during contact, practical and workplace time for this programme – make sure that you have his/her contact details.
- ✓ You are responsible for the safekeeping of your completed Formative Assessment Workbook and Workplace Guide
- ✓ If you need assistance please contact your facilitator who will gladly assist you.

- ✓ If you have any special needs please inform the facilitator

Learner Expectations

Please prepare the following information. You will then be asked to introduce yourself to the instructor as well as your fellow learners

Your name:
The organisation you represent:
Your position in organisation:
What do you hope to achieve by attending this course / what are your course expectations?

UNIT STANDARD 9010

Unit Standard Title

Demonstrate an understanding of the use of different number bases and measurement units and an awareness of error in the context of relevant calculations

NQF Level

3

Credits

2

Purpose

This Unit Standard is intended to provide credits towards the mathematical literacy requirements of the NQF at level 2. The essential purposes of the mathematical literacy requirements are that, as the learner progresses with confidence through the levels, the learner will grow in:

- ✓ An insightful use of mathematics in the management of the needs of everyday living to become a self-managing person
- ✓ An understanding of mathematical applications that provides insight into the learner `s present and future occupational experiences and so develop into a contributing worker
- ✓ The ability to voice a critical sensitivity to the role of mathematics in a democratic society and so become a participating citizen.

People credited with this unit standard are able to:

- ✓ Convert numbers between the decimal number system and binary number system
- ✓ Work with numbers in different ways to express size/magnitude.
- ✓ Demonstrate the effect of error in calculations.

Learning Assumptions

Learners accessing this Unit Standard should be competent in Mathematical Literacy and Communications at NQF level 2.

Specific Outcomes and Assessment Criteria

Specific Outcome 1: Convert numbers between the decimal number system and the binary number system: This outcome includes the need to: Perform addition and subtraction of positive whole numbers in binary up to 100002 (16 in decimal), Demonstrate understanding of the mathematical relationships and principles involved in the computations.

Assessment Criteria

- ✓ Conversion between binary and decimal numbers is done correctly

- ✓ Basic addition and subtraction calculations in the binary number system are done correctly
- ✓ Using positive whole numbers up to the 16 in decimal
- ✓ Practical applications of the decimal and binary system are explained correctly

Specific Outcome 2: Work with numbers in different ways to express size and magnitude: This outcome includes the need to use scientific notation for small and large numbers.

Assessment Criteria

- ✓ The prefixes indicating magnitude in measurements are correctly related to the decimal system.
- ✓ The prefixes indicating magnitude in measurements are correctly related to the decimal system: From Giga to Pica (1012 to 10 -12)
- ✓ Conversions between related units in different measurement systems are correctly applied in real-life contexts: SI to Imperial; Degrees F to degrees C

Specific Outcome 3: Demonstrate the effect of error in calculations

Assessment Criteria

- ✓ Symbols for irrational numbers such as 7c and 42 are left in formulae or steps to calculations except where approximations are required
- ✓ Descriptions are provided of the effect of rounding prematurely in calculations
- ✓ The desired degree of accuracy is determined in relation to the practical context
- ✓ The final value of a calculation is expressed in terms of the required unit

Unit Standard Essential Embedded Knowledge

- ✓ Number systems and rational and irrational numbers
- ✓ Estimation and approximation
- ✓ Scientific notation

Critical cross-field outcomes

Upon successful completion of this course, the learner will be able to:

- ✓ Collect, analyse, organise and critically evaluate information: Gather, organise, and interpret numerical information.
- ✓ Communicate effectively: Use everyday language and mathematical language to describe relationships, processes and problem solving methods.
- ✓ Use mathematics: Use mathematics to describe and represent realistic situations and to solve problems relevant to the learner

DECIMAL AND BINARY SYSTEM

Outcome

Convert numbers between the decimal number system and the binary number system:

Assessment criteria

- ✓ Conversion between binary and decimal numbers is done correctly
- ✓ Basic addition and subtraction calculations in the binary number system are done correctly
- ✓ Using positive whole numbers up to the 16 in decimal
- ✓ Practical applications of the decimal and binary system are explained correctly

Computational Tools

A sequence of calculations that sets out a series of detailed steps enabling a particular result to be obtained, is called an algorithm. The process of long division, for instance, is an algorithm.

The value of algorithms has been rediscovered with the development of calculating machines. These machines are used in many forms today, varying from pocket calculators to complicated computers.

For the purpose of this course the simple pocket calculator will be discussed as a computational tool.

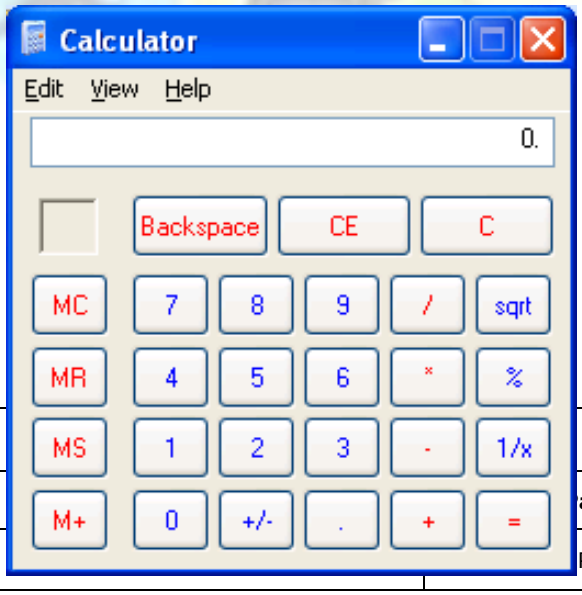
The liquid crystal display (LCD) of the calculator is made of a liquid crystal, hermetically sealed between two glass plates and caution must be exercised in handling the calculator.

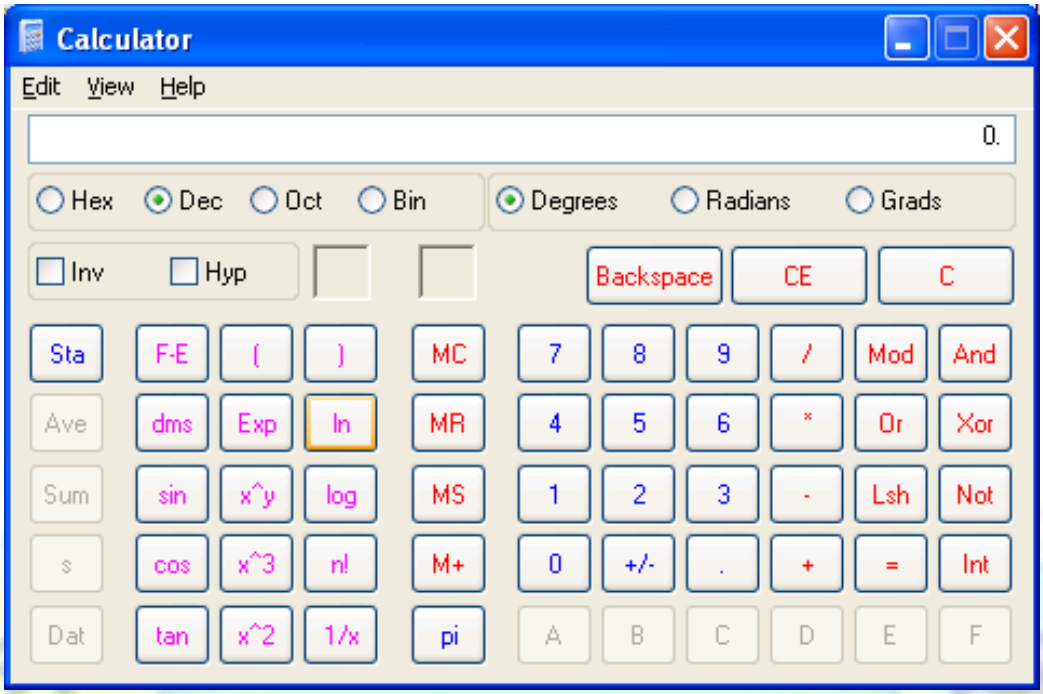
The following general rules must be kept in mind to ensure trouble free operation:

- ✓ Do not place the calculator in a location subject to direct sunlight, especially in a car with its windows closed in a hot climate. High temperatures may damage the calculator.
- ✓ Avoid locations subject to rapid temperature changes and excessive moisture or dust.
- ✓ Do not drop or bump the calculator.
- ✓ Always use a soft dry cloth to clean the calculator – not a cloth moistened with any volatile solvent or water.
- ✓ If the calculator uses batteries, do not leave the batteries in it for extended periods if the unit is not to be used. Battery acid leakage may damage the calculator.

A wide variety of calculations can be done, depending on the type of calculator used. Specialised calculators are used for example scientific, statistic or financial purposes.

It is a good idea to study the manual of your specific calculator well to be able to maximize the use of your computational tool





Introduction to numbers and algorithms

Addition and Subtraction

Example 1

123 + 456 + 789

Key in: 123 + 456 + 789

Answer: 1368

Example 2

100 - 25 - 35

Key in: 100 - 25 - 35

Answer: 40

Pressing the = key gives the answer to the entered formula.

Formative Assessment

Use appropriate algorithms to do the following calculations on your calculator:

Find the sum of:

52	68	
1345	65	
15.8	989	
1123.4	6598	
12.894	1345	
12.368	15.8	
0.0012	1123.4	
68	12.894	
65	12.368	
989	0.0012	
6598	15.8	

John is a bus driver and travelled the following distances during his first trip for the day:

Bus stop	**Arrive**	**Depart**	**Distance**
A	6:15	6:26	37 km
B	6:38	6:44	8 km
C	7:01	7:13	11 km
D	7:36	7:47	19 km
E	8:07	8:19	9 km

How far did he travel from A to C?

How far did he travel from C to D?

What was the total distance covered in this trip?

How long did he spend at bus stop A?

How long did he spend at stops C and D?

Multiplication and Division

Example 3

50 x (-2) ÷ 4

Key in: 50 x 2 ± ÷ 4 =

Answer: -25

NOTE: To enter a negative number, press the ± (change Sign) key after numeric entry.

Example 4

5 + 2 x 3 - 2 ÷ 0.5

Key in: 5 + 2 x 3 -2 ÷ 1 =

Answer: 7

Parentheses (Brackets)

The parentheses key are used to cluster together a series of operations if it is necessary to do it first. When brackets are used the calculations in brackets take precedence over any other calculation. Calculations within the innermost set of brackets will be performed first.

Example 5

12 + 42 ÷ (8 - 6)

Key in: 12 + 42 ÷ (8 - 6) =

Answer: 33

Example 6

(3 + 4) x (3 - 1)

Key in: (3 + 4) x (3 - 1) =

Answer: 24

Important: An error will occur should the brackets be omitted!

Formative Assessment

Calculate the answers:

(3.56 x 2.34) +(2.3 – 1.2)

(11.2 – 5.6) – (2.4 + 4.3)

(989.21 – 3.4) x (5.3 – 2.3)

10.99 + (7.8 x 2.2)

414.3 – 298.99 +3.56

42.2 x(5.3 – 4.3)

33.1 x(4.5 + 3.9)

0.003 + 2.13 x (4.5 + 4.2)

Determine the error should you omit the brackets.

Integers and real numbers

Integers are whole numbers (e.g. 1, 45, 77…) and ***real numbers*** are numbers with a decimal point (e.g. 24.59, 2.09 and 9.1)

Note that a whole number is *real* if it is written with a decimal point. Therefore;

64 is an integer, but

64.0 is real

Number Systems

A numeral is a symbol or group of symbols, or a word in a natural language that represents a number. Numerals differ from numbers just as words differ from the things they refer to. The symbols "11", "eleven" and "XI" are different numerals, all representing the same number. This article attempts to explain the various systems of numerals.

A numeral system (or system of numeration) is a framework where a set of numbers are represented by numerals in a consistent manner. It can be seen as the context that allows the numeral "11" to be interpreted as the binary numeral for three, the decimal numeral for eleven, or other numbers in different bases.

Decimal

There is more than one number system. The decimal number system is the most common where there are 10 elements

Elements:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Base Number:

10

The decimal system is the one we use to count and perform operations on numbers. Perhaps the decimal system is used because humans have ten fingers and counting objects began with using our ten digits, as they are called. I wonder how we would be counting if we had eight or twelve fingers. I suppose we will never know.

When the decimal system is mentioned, almost everyone says that they know and understand the system. However, when asked to count, they start at 1 and count to 10. This is incorrect! The decimal system consists of ten items but they actually start at 0 and end at 9.

The Arabic nations invented a method to represent two extremely important concepts:

- ✓ how to represent zero and
- ✓ a system so that each number in a specific position has a well-known and easily definable meaning or weight.

It may sound strange but the invention of zero (or nothing) was the biggest breakthrough in mathematical history. together with the fact that the position of each digit had a specific meaning meant that commerce and trade were simplified. The western world uses slight modifications of the numbering figures invented so long ago by the Arabic nations.

Hierarchy of Decimal Numbers

Number	***Name***	***How many***
0	zero	
1	one	One
2	two	two ones
3	three	Three ones
4	four	Four ones
5	five	Five ones
6	six	Six ones
7	seven	seven ones
8	eight	Eight ones
9	nine	Nine ones
10	ten	one ten
20	twenty	two tens
30	thirty	three tens
40	forty	four tens

50	fifty	five tens
60	sixty	six tens
70	seventy	seven tens
80	eighty	eight tens
90	ninety	nine tens
Number	***Name***	***How Many***
100	one hundred	ten tens
1,000	one thousand	ten hundreds
10,000	ten thousand	ten thousands
100,000	one hundred thousand	one hundred thousands
1,000,000	one million	one thousand thousands

Some people use a comma to mark every 3 digits. It just keeps track of the digits and makes the numbers easier to read.

The Binary Digit System

The binary digit system is very important to our daily life, as all information contained inside computers consist of it. "Bi" means two, and that is the basis of this whole numerical system.

It consists of the basic elements 0 and 1.

There is no difference between the way the binary system and the decimal system consider 0 and 1. But how do we represent 2 in the binary system? The answer is the same as for any other numerical system. When the single digits are all used we move on to double digits.

Elements:

0, 1

Base Number:

2

Binary Representation Of The Decimal Numbers 1 To 35:

Decimal value	Binary value	Decimal value	Binary value	Decimal value	Binary value
0	0	16	110	32	100000
1	1	17	111	33	100001
2	10	18	1000	34	100010
3	11	19	1001	35	100011
4	100	20	1010		
5	101	21	1011		
6	110	22	1100		
7	111	23	1101		
8	1000	24	1110		
9	1001	25	1111		
10	1010	26	10000		
11	1011	27	10001		
12	1100	28	10010		
13	1101	29	10011		
14	1110	30	10100		
15	1111	31	10101		

Converting From Binary Digits To Decimal Numbers

Show the extended notation of 11010

Example:

$11010_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

Example

Express the binary value 1011011001 as a decimal number.

1011011001

$=1x2^9 + 0x2^8 + 1x2^7 + 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0$

$=1x2^9 + 1x2^7 + 1x2^6 + 1x2^4 + 1x2^3 + 1x2^0$

=512 +128 + 64 + 16 + 8 +1

$= (729)_1$ or

$(1011011001)_2 = (729)_{10}$

Conversion Of A Decimal Number To A Binary Number

Convert the decimal number 25 to a binary number

Example

Here you repeatedly divide by 2

25	
12	1
6	0
3	0
1	1
0	0
	11001

Addition Of Binary Numbers

$(1011 + 1111)_2$

Example

	1	1	1(with1 Carry over)	(with 0 Carry over)
	1	0	1	1
	1	1	1	1+
1	1	0	1	0
2+	3 2+	2 2+	3 2+	2
	10	10	10	10
	1 with 1	1 with 0	1(with1 Carry over)	1 (with 0 Carry over)

$(1011 + 1111)_2 = 11010$

Subtraction Of Binary Numbers

$(11000 – 10101)_2$

11000

-10101

00011

$(11000 – 10101)_2 = 00011$

Formative assessment

Question 1: Convert the following decimal numbers to binary numbers:

a	3	
b	5	

c	7	
d	11	

Question 2: Convert the following binary numbers to decimal numbers

a	1	
b	10	
c	100	
d	101	

Question 3: Add the following binary numbers and convert the answers to decimal

a	10 and 11		
b	101 and 11		
c	100 and 101		
d	111011 and 100001		

Question 4: Subtract the following binary numbers and convert the answers to decimal

a	10 – 1		
b	111 – 11		
c	110 – 11		
d	110 – 10		

EXPRESS SIZE AND MAGNITUDE

Outcome

Work with numbers in different ways to express size and magnitude

Assessment criteria

- ✓ The prefixes indicating magnitude in measurements are correctly related to the decimal system: From Giga to Pica (1012 to 10 -12)
- ✓ Conversions between related units in different measurement systems are correctly applied in real-life contexts: SI to Imperial; Degrees F to degrees C

Express Size And Magnitude Using Numbers

What do you notice from the following two paragraphs?

The earth is approximately 149,503,000km from the sun and is moving through space at approximately 72,360km/h towards the constellation Hercules. The approximate length of earth's orbit around the sun is 938,900,000km and the earth travels on it at about 106,000km/h.

All matter including the earth consists of atoms. The diameter of a typical nucleus of an atom is approximately 0.00000000000001 metre or about 0.00001 times the diameter of the atom.

In the first paragraph you should notice that rounding took place (the zeros at the end of each number usually indicate rounding). In the second paragraph you should notice that very small numbers are difficult to understand and interpret. Are there better ways to represent these numbers?

The answer is that there are better ways to represent these numbers and to work with them as well. This means that numbers can be used in different ways to express size and magnitude.

SI Units

South Africa uses the SI system of measurements, as do most countries of the world. International System of Units (French Le Système International d'Unités) is commonly referred to throughout the world as SI, after the initials of Système International

The SI or ***Systéme International*** consists of 7 base units, which were taken into use in order to have a worldwide acknowledged unit system. This has significantly simplified the sharing of information between countries with different traditional units.

Quantity	***Unit***	***Symbol***
Mass	Kilogram	kg
Length	Meter	m
Time	Second	s

Temperature	Kelvin	K
Current	Ampére	A
Light	Candela	cd
Chemical standard unit	Mole	mol

It is VERY important to always indicate a unit. The unit is what gives meaning to a number. Just think 3000 tells you nothing about what this number is for or what it does, but R3000 is very useful! Also remember to indicate the unit EXACTLY as it is shown above. Km is wrong and so is S, if the unit is not given exactly right your answer will be wrong!

The symbols in the last column are not abbreviations (hence, no periods are used), and they are exactly the same in all languages. Prefixes may be added to these symbols in order to conveniently refer to very large or very small quantities. The prefixes are listed in below.

Prefix	Symbol	Factor	Power of 10
Exa	E	1,000,000,000,000,000,000	18
Peta	P	1,000,000,000,000,000	15
Tera	T	1,000,000,000,000	12
Giga	G	1,000,000,000	9
Mega	M	1,000,000	6
Kilo	k	1,000	3
Hecto	h	100	2
Deka	da	10	1
Deci	d	0.1	-1
Centi	c	0.01	-2
Milli	m	0.001	-3
Micro	µ	0.000001	-6
Nano	n	0.000000001	-9
Pico	p	0.000000000001	-12

Femto	f	0.000000000000001	-15
Atto	a	0.000000000000000001	-18

SI PREFIXES, SYMBOLS AND FACTORS

These prefixes are used for every unit (supplementary or derived) with the exception of the kilogram.

Examples are millimetre (mm), kilometre/hour (km/h), megawatt (MW), and picofarad (pF). Because double prefixes are not used, and because the base unit kilogram already contains a prefix, prefixes are not used with kilogram, although they are used with gram.

The prefixes hecto, deka, deci and centi are used only rarely, and then usually with metre to express areas and volumes. Because of established usage, the centimetre is retained for body measurements and clothing.

Some examples of SI derived units:

Quantity	Name of derived SI unit	Symbol
Area	square metre	m2
Volume	cubic metre	m3
Velocity	metre per second	m/s
Acceleration	metre per second squared	m/s2
Density	kilogram per cubic metre	kg/m3
Current density	ampere per square metre	A/m2
Magnetic field strength	ampere per metre	A/m
Specific volume	cubic metre per kilogram	m3/kg
Luminance	candela per square metre	cd/m2

The metre and the kilogram had their origin in the metric system. The metre is defined as the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second.

When the metric system was created, the kilogram was defined as the mass of 1 cubic decimetre of pure water at the temperature of its maximum density (4.0°C). Today the reference kilogram is a platinum-iridium cylinder.

Converting between units is straightforward as these examples show. If you don't understand these examples, refer back to the table containing symbols and factors. Remember that you are only moving a decimal point and changing a name. You don't have any real arithmetic to do.

1kg (kilogram) = 1,000g (grams) = 100dag (dekagrams) = 10hg (hectograms).

1kl (kilolitre) = 1,000l (litres) = 100dal (decilitres) = 10hl (hectolitres).

Although the values of the factors differ by multiples of 10, some symbols and names are rarely used. Of the six symbols and names in this example, I have only seen kg (kilogram), g (gram) and l (litre). So let's look at some common uses.

If I travel 1,000km I say: 'I travelled 1,000kms.' I don't say: 'I travelled 1Mm (megametre)'. It is not incorrect to use megametre for 1,000km but nobody I know uses that phrase. However, you may see a few other uses that at first glance appear strange. The contents of bottles may contain 750ml or 75dl but not usually 0.75l and medication may contain 600mg of a substance but not usually 0.6g. These values refer to the same measurements (750ml = 75dl = 0.75l and 600mg = 0.6g).

Using the SI system

The SI system uses the metric (decimal) system and uses a number of standard prefixes for units of length and mass that were covered in the previous section. Using the SI system means that we should know the most important ones. The three most important ones are:

$$\boxed{\begin{aligned} \text{kilo} &= 1000 \\ \text{centi} &= \frac{1}{100} \\ \text{milli} &= \frac{1}{100} \end{aligned}}$$

Formative assessment

Complete each of the following:

1. 150cm = _____m
2. 360mm = _____m
3. 62ml = _____litres
4. 3.6 tonnes = _____kg

Combinations of base units

Many of the special units used in physics are combinations of these base units.

Example

Force: $kg.m.s^{-2}$ also Newton

Multiples and fractions commonly used:

Prefix	***Abbreviation***	***Value***
Giga	G	10^{9}
mega	M	10^{6}
Kilo	k	10^{3}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}

Units of time

Note that one year consists of 12 months but the month has not been defined. As an approximation, a month consists of 30 days and 22 workdays. For calculations of intervals less than one week the second is accurate and may be used. However, the second is rarely used for intervals greater than one day.

The relationship of a week, day, hour and minute calculated in terms of seconds:

Unit	**Calculation to seconds**
1 minute	60 seconds
1 hour	60 minutes 3600 seconds (60 × 60)
1 day	24 hours 1,440 minutes (24 × 60) 86,400 seconds (24 × 60 × 60)

1 week	7 days 168 hours (7 × 24) 10,080 minutes (7 × 24 × 60) 604,800 seconds (7 × 24 × 60× 60)

THE WEEK, DAY, HOUR AND MINUTE RELATED TO SECONDS

Speed

The lists units in common use for speed and their abbreviations.

Distance	Time	Speed	Abbreviation
kilometres	hours	kilometres per hour	km/h
metres	hours	metres per hour	m/h
metres	seconds	metres per second	m/s
centimetres	seconds	centimetres per second	cm/s or cm/sec

The abbreviation 'cm/sec' is not officially correct but it is found in practice. The other abbreviations follow the units specified by SI system.

Scientific Notation

Scientific notation is very useful as it allows you to express very large or small numbers easily and is also helpful in preventing ambiguity in regard to significant figures.

For example: 1000 could be construed as having 1 significant figure or 4, however if it is indicated as 1×10^3 we know it only has 1 or $1{,}000 \times 10^3$ and we know it has 4.

Examples

1 km = 1 x 1000 m = 1×10^3m

12345m=$1{,}2345 \times 10^4$m

0,000012m = $1{,}2 \times 10^{-5}$m

0,12m = $1{,}2 \times 10^{-1}$m

1mm = 1/1000m = 1×10^{-3}m

Using Scientific Notation

Calculate $(1{,}234 \times 10^6) + (6{,}7 \times 10^3)$

Solution:

$(1{,}234 \times 10^6) + (6{,}7 \times 10^3)$

$= (1{,}234 \times 10^6) + (0{,}0067 \times 10^6)$

$= (1{,}234 + 0{,}0067) \times 10^6$

$= 1{,}2407 \times 106$

$\approx 1{,}241 \times 10^6$

Imperial Units

The Imperial units are also used in certain countries. The imperial system, now called the UK system, was used, until very recently, for all weights and measures throughout the UK.

The UK system measurement of length:

Length		
12 inches	=	1 foot
3 feet	=	1 yard
22 yards	=	1 chain
10 chains	=	1 furlong
8 furlongs	=	1 mile
5280 feet	=	1 mile
1760 yards	=	1 mile

The UK system for area.

Area		
144 sq. inches	=	1 square foot
9 sq. feet	=	1 square yard
4840 sq. yards	=	1 acre
640 acres	=	1 square mile

The UK system for volume

Volume

1728 cu. inches	=	1 cubic foot
27 cu. feet	=	1 cubic yard

The UK system for capacity.

Capacity		
20 fluid ounces	=	1 pint
4 gills	=	1 pint
2 pints	=	1 quart
4 quarts	=	1 gallon (8 pints)

The UK system for mass (avoirdupois)

Mass (Avoirdupois)		
437.5 grains	=	1 ounce
16 ounces	=	1 pound (7000 grains)
14 pounds	=	1 stone
8 stones	=	1 hundredweight [cwt]
20 cwt	=	1 ton (2240 pounds)

The following are a few practical estimations from the SI system that you may find useful in your daily duties:

1. Most adults are between 1.5 and 1.8 metres tall.
2. The length of an adult pace is about 1 metre.
3. It takes about 15 minutes to walk one kilometre.
4. The height of a standard door is about 2 metres.
5. The mass of a family car is about one tonne (1,000kg).
6. The mass of a bag of sugar is about 1kg.
7. One hectare = 10,000m^2 or about two football pitches.
8. The volume of a normal tin of drink is about 330cm^2.
9. A teaspoon holds about 5ml of liquid.

In a like manner, the following are a few practical estimations from the UK system that you may find useful

1. The height of a tall adult is about 6 feet.
2. The width of an adult thumb is about 1 inch.
3. The length of a size 8 shoe is about 1 foot.
4. An adult pace is about 1 yard.
5. The mass of a bag of sugar is just over 2 pounds.
6. An old-style bottle of milk contains 1 pint.
7. It takes about 20 minutes to walk one mile.

Commonly used unit conversions to convert between Imperial and SI

1 inch (in.) = 2.54 cm
1 cm = 0.3937 in.
1 foot (ft) = 30.48 cm
I m = 39.37 in. = 3.281 ft
1 mile (mi) = 5280 ft = 1.609 km
1 km = 0.6214 mi
1 gallon (gal U.S.) = 3.785 L = 0.8328 gal(British)
1 pint (British) = 568 mL
1mi/h = 1.467 ft/s = 1.609 km/h
1km/h = 0.287 m/s = 0.621 mi/h
1 ft/s = 0.305 m/s
1m/s = 3.281 ft/s= 3.600 km/h
I kg = 2.20 pounds (lb)

Formative assessment

Complete the following table

Quantity	Unit	Abbreviation
Mass		

		m
		s
Temperature		
	Ampére	
Light		
Chemical standard unit		

Question2: Write the following in scientific notation:

a	0.0009 m	9×10^{-4} m
b	104869.4572 l	$1{,}048694572 \times 10^{5}$ l
c	34,140089 m	$3{,}4140089 \times 10^{1}$ m
d	395867.5264 cm	$3.958675264 \times 10^{5}$ cm
e	367.666 km	3.67666×10^{2} km
f	444444.444 mm	4.44444444×10^{5} mm
g	0,00000000089 ml	$8{,}9\ 10^{-10}$ ml

<u>Question 3:</u> Complete the table below:

Prefix	**Abbreviation**	**Value**
	G	10^{9}
	M	10^{6}
	k	10^{3}
	m	10^{-3}
	μ	10^{-6}

	n	10^{-9}
	p	10^{-12}

Question 4: Convert the units in the table. Remember to show your calculations

12 lb to kg	
100 ft to m	
300 cm to in.	
23 ft/s to m/s	
6 pint to L	
156 mm to m	

Temperature Scales

There are three commonly used temperature scales:

- ✓ The Celsius scale is the most commonly used temperature scale.
- ✓ The Fahrenheit scale is used in the United States.
- ✓ The absolute or Kelvin scale is used in scientific work.

The Fahrenheit and Celsius scales assign arbitrary values to both freezing and boiling points of water at atmospheric pressure.

	Celsius	***Fahrenheit***
Freezing point	0.00°C	32.0°F
Boiling point	100°C	212°F

Between these two reference points the Celsius scale is divided into 100 equal units and the Fahrenheit scale into 180 equal units. This makes it easy to convert from Celsius to Fahrenheit or vice versa, as each value of Celsius has a corresponding Fahrenheit value, 1°F = 9/5°C. The conversion formulas are as follows:

T(°C) = 9/5[T(°F)-32] or T(°F) = 9/5T(°C) + 32

It may be easier to simply remember that 0°C=32°F and that 5°C=9°F.

Example: Taking Your Temperature

Normal body temperature is 98.6°F. What is this in °C? And what is the temperature in Fahrenheit back from °C?

Solution:

98.6-32) x 9/5 = 37°C

37 x 9/5 + 32 = 98.6 °F

Convert From Fahrenheit To Celsius Using The Formulae

T(°C) = 9/5[T(°F)-32]

°F = 98.6

°C = ?

Substitute in formulae above

T(°C) = 9/5 x [98.6 – 32]

T(°C) = 9/5 x [66.6]

T(°C) = 37°C

Convert Fahrenheit into Celsius using the formula:

T(°F) = 9/5T(°C) + 32

°C = 37

°F = ?

Substitute in formulae given above

T(°F) = 9/5 x 37°C + 32

T(°F)= 66.6°C + 32

T(°F) =98.6 °F

It is important to remember that different thermometers are made from various materials and filled with different substances, in practice this means that they all expand and contract differently in response to changes in temperature. Because of this most thermometers are only reliable within a set range of temperatures.

EFFECT OF ERROR IN CALCULATIONS

Outcome

Demonstrate the effect of error in calculations

Assessment criteria

- ✓ Symbols for irrational numbers such as 7c and 42 are left in formulae or steps to calculations except where approximations are required
- ✓ Descriptions are provided of the effect of rounding prematurely in calculations
- ✓ The desired degree of accuracy is determined in relation to the practical context
- ✓ The final value of a calculation is expressed in terms of the required unit

Groups Of Numbers

Not all numbers are exact – some are actually infinite. This makes it difficult or impossible to represent them exactly. The main groups of numbers will be discussed below.

All Real numbers, (numbers with real values) can be divided into Rational and Irrational numbers.

Rational numbers have values that can be determined exactly and Irrational numbers have values that cannot be exactly determined. Rational numbers are very often perfect squares.

Whole numbers, or integers, suffice for discussing numbers of people or cattle, but entities land, wine and grain often need to be measured out in varying quantities that does not correspond to whole numbers. Among the followers of Pythagoras it was discovered that whole numbers and fractions (known as rational numbers because they can be expressed as ratios) do not account for all numbers.

The numbers that can not be expressed as ratios are called irrational. An example is the square root of 2 (the number that multiplied by itself equals 2). Rational and irrational numbers together represent all numbers greater than zero.

It was not until the Renaissance that the progress of mathematics called for a further extension of the numbers below zero. It was gradually realised that these "negative" numbers were an acceptable mathematical idea, provided that they are handled consistently.

The concept of negative numbers occurs in every day life. A person can not find a negative number of money in his pocket, but his overdraft at the bank can be negative.

Rational numbers are all numbers that can be represented as a ratio $\left(\frac{a}{b}\right)$ of two numbers. All whole numbers are rational numbers because they may be represented as their value over 1. The number 3 is therefore $\frac{3}{1}$.

Irrational numbers are those that cannot be represented as a ratio of two whole numbers. Two irrational numbers are $\sqrt{2}$ and π. When dealing with arithmetic involving irrational numbers it is best to keep them as they appear unless an approximate answer is required.

Real numbers	
Rational numbers	Irrational numbers
⅔; √4; √9; √(16/25)	√5; $\sqrt[3]{7}$

Fractions

Digits to the right of the decimal point represent the fractional part of the decimal number. Each place value has a value that is one tenth the value to the immediate left of it.

Number	Name	Fraction
.1	tenth	1/10
.01	hundredth	1/100
.001	thousandth	1/1000
.0001	ten thousandth	1/10000
.00001	hundred thousandth	1/100000

Example

0.234 = 234/1000 (said - point 2 3 4, or 234 thousandths, or two hundred thirty four thousandths)

4.83 = 4 83/100 (said - 4 point 8 3, or 4 and 83 hundredths)

Repeating decimals are decimals with certain digits that repeat.

Example: 1.3333333 never ends and is written as:

$$1.33333..... = \frac{1}{3}$$

However this is a rather complicated method to use. It is much easier to simplify the repeating number to its nearest decimal value. We do however indicate that it is repetitive.

$$1.33333..... = 1.\dot{3} = 1\frac{3}{10}$$

Here are a few examples:

$$1.454545..... = 1.\dot{4}\dot{5} = 1\frac{45}{100}$$

$$2.456456456..... = 2.\dot{4}\dot{5}\dot{6} = 2\frac{456}{1000}$$

Often when we calculate a number we end up with a lot of decimals, this isn't practical and we round off to a certain number of decimal numbers.

If a number is between 1 - 4 it is rounded down and if it is between 5-9 it is rounded up.

Example

Round off all the numbers to 3 decimal numbers

1.256784 = 1.257

4.3812629 = 4.381

1.001111 = 1.001

22.22222 = 22.222

8.989993 = 8.990

The Effect Of Error In Calculations

From the explanation it can be seen that not all answers are hundred percent correct even if our arithmetic was correct. It depends on more than that. Often we use values that were measured and these can never be perfectly accurate.

Significant Figures

Significant figures help us to work as accurately as possible when working with measured data. A 2 figure measurement, like 91 m is accurate to ±1 m.

The percentage error is therefore:

1/91 x 100 = 1, 09% ≈1%

A 4 figure measurement like 91,11m is accurate to 0,01m. The percentage error is therefore:

0, 01/91.11 x 100 = 0, 0109 ≈0, 01

This is 100 times more accurate!

When trying to determine the number of significant figures a number has you should not consider zeros before or after the number. Unless the number has a full stop at the end, this is common in American notation.

For Example

10 has 1 significant figure

0,00234 has 3 significant figures

2340. has 4 significant figures

3.45 has 3 significant figures

Rules Used To Determine The Number Of Significant Figures The Answer Of A Calculation Should Have

Multiplication and division:

Number with the least significant figures determines the answer:

12,345 x 6, 7 = 83 NOT 82, 7115

Addition and subtraction

Retain the smallest number of decimal places.

10,345 + 9, 9 = 20.2 NOT 20,245 or 20!

Convert repeating decimals to common fraction form

Important Note: any span of numbers that is underlined signifies that those numbers are repeated. For example, 0.09 signifies 0.090909.

Only fractions in lowest terms are listed. For instance, to find 2/8, first simplify it to 1/4 then search for it in the table below.

fraction = decimal			
1/1 = 1			
1/2 = 0.5			
1/3 = 0.3	2/3 = 0.6		
1/4 = 0.25	3/4 = 0.75		
1/5 = 0.2	2/5 = 0.4	3/5 = 0.6	4/5 = 0.8
1/6 = 0.16	5/6 = 0.83		
1/7 = 0.142857	2/7 = 0.285714	3/7 = 0.428571	4/7 = 0.571428

	5/7 = 0.714285	6/7 = 0.857142	
1/8 = 0.125	3/8 = 0.375	5/8 = 0.625	7/8 = 0.875
1/9 = 0.1	2/9 = 0.2	4/9 = 0.4	5/9 = 0.5
	7/9 = 0.7	8/9 = 0.8	
1/10 = 0.1	3/10 = 0.3	7/10 = 0.7	9/10 = 0.9
1/11 = 0.09	2/11 = 0.18	3/11 = 0.27	4/11 = 0.36
	5/11 = 0.45	6/11 = 0.54	7/11 = 0.63
	8/11 = 0.72	9/11 = 0.81	10/11 = 0.90
1/12 = 0.083	5/12 = 0.416	7/12 = 0.583	11/12 = 0.916
1/16 = 0.0625	3/16 = 0.1875	5/16 = 0.3125	7/16 = 0.4375
	11/16 = 0.6875	13/16 = 0.8125	15/16 = 0.9375
1/32 = 0.03125	3/32 = 0.09375	5/32 = 0.15625	7/32 = 0.21875
	9/32 = 0.28125	11/32 = 0.34375	13/32 = 0.40625
	15/32 = 0.46875	17/32 = 0.53125	19/32 = 0.59375
	21/32 = 0.65625	23/32 = 0.71875	25/32 = 0.78125
	27/32 = 0.84375	29/32 = 0.90625	31/32 = 0.96875

Formative assessment

Question1: Convert the following repeating decimals to common fractions:

d	1.45454545	
e	1,33333	
f	52.535535535	
g	909.9090909090	

Question 2: Convert into °C:

a	10°F	
b	38°F	
c	100°F	

Convert into °F:

a	100 °C	
b	50 °C	
c	66 °C	

UNIT STANDARD 9013

Unit Standard Title

Describe, apply, analyse and calculate shape and motion in 2-and 3-dimensional space in different contexts.

Unit Standard ID

9013

NQF Level

3

Credits

4

Purpose

This unit standard is designed to provide credits towards the mathematical literacy requirements of the NQF at level 3. The essential purposes of the mathematical literacy requirements are that, as the learner progresses with confidence through the levels, the learner will grow in:

- ✓ An insightful use of mathematics in the management of the needs of everyday living to become a self-managing person
- ✓ An understanding of mathematical applications that provides insight into the learner's present and future occupational experiences and so develop into a contributing worker
- ✓ The ability to voice a critical sensitivity to the role of mathematics in a democratic society and so become a participating citizen.

People credited with this unit standard are able to:

- ✓ Measure, estimate, and calculate physical quantities in practical situations relevant to the adult in life or the workplace
- ✓ Explore describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations

Learning Assumed To Be In Place

The credit value is based on the assumption that people starting to learn towards this unit standard are competent in Mathematical Literacy and Communications at NQF level 2.

Unit Standard Range

- ✓ The scope of this unit standard includes length, surface area, volume, mass, speed ; ratio and proportion; making and justifying conjectures.
- ✓ Contexts relevant to the adult, the workplace and the local community.

- ✓ More detailed range statements are provided for specific outcomes and assessment criteria as needed.

Specific Outcomes And Assessment Criteria

Specific Outcome 1: Measure, estimate, and calculate physical quantities in practical situations.

Outcome Notes:

Measure, estimate, and calculate physical quantities in practical situations relevant to the adult in life or the workplace.

Outcome Range:

- ✓ Basic instruments to include those readily available such as rulers, measuring tapes, measuring cylinders or jugs, thermometers, spring or kitchen balances, watches and clocks.
- ✓ In situations which necessitate it such as in the workplace, the use of more accurate instruments such as vernier calipers, micrometer screws, stop watches and chemical balances.
- ✓ Quantities to estimate or measure to include length/distance, area, mass, time, speed and temperature.
- ✓ Estimate the area and volume of simple irregular shapes and objects.
- ✓ The quantities should range from the low or small to the high or large.
- ✓ Mass, volume temperature, distance, and speed values are used in practical situations relevant to the learner or the workplace.
- ✓ Calculations involving the effects on area and volume when altering linear dimensions.
- ✓ Calculate heights and distances using Pythagoras` theorem.
- ✓ Calculate surface areas and volumes of right prisms (i.e., end faces are polygons and the remaining faces are rectangles) and cylinders from measurements in practical situations relevant to the life of the learner or in the workplace.

Assessment Criteria

- ✓ Scales on the measuring instruments are read correctly.
- ✓ Quantities are estimated to a tolerance justified in the context of the need.
- ✓ The appropriate instrument is chosen to measure a particular quantity.
- ✓ Quantities are measured correctly to within the least step of the instrument.
- ✓ Calculations are carried out correctly.
- ✓ Symbols and units are used in accordance with SI conventions and as appropriate to the situation.

Specific Outcome 2: Explore, describe and represent, interpret and justify geometrical relationships and conjectures**.**

Outcome Notes:

Explore, describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations.

Outcome Range:

- ✓ Applications taken from different contexts such as packaging, arts, building construction, dressmaking.
- ✓ The use of tessellations and symmetry in artifacts and in architecture.
- ✓ Use rough sketches to interpret, represent and describe situations.
- ✓ Use and interpret scale drawings of plans (e.g., plans of houses or factories; technical diagrams of simple mechanical household or work related devices such as jacks,
- ✓ Nets of prisms and cylinders.
- ✓ Road maps relevant to the local community.
- ✓ The use of the Cartesian co-ordinate system in determining location and describing relationships in at least two dimensions.

Assessment Criteria:

- ✓ Descriptions are based on a systematic analysis of the shapes and reflect the properties of the shapes accurately, clearly and completely.
- ✓ Descriptions include quantitative information appropriate to the situation and need.
- ✓ Conjectures as appropriate to the situation, are based on well-planned investigations of geometrical properties.
- ✓ Representations of the problems are consistent with and appropriate to the problem context. The problems are represented comprehensively and in mathematical terms.
- ✓ Results are achieved through efficient and correct analysis and manipulation of representations.
- ✓ Problem-solving methods are presented clearly, logically and in mathematical terms.
- ✓ Solutions are correct and are interpreted and validated in terms of the context of the problem.

Unit Standard Essential Embedded Knowledge

The following essential embedded knowledge will be assessed through assessment of the specific outcomes in terms of the stipulated assessment criteria. Candidates are unlikely to achieve all the specific outcomes, to the standards described in the assessment criteria, without knowledge of the listed embedded knowledge. This means that the possession or lack of the knowledge can be inferred directly from the quality of the candidate's performance against the standards.

- ✓ Properties of geometric shapes
- ✓ Length, area, volume, mass, time, temperature, speed
- ✓ The Cartesian system
- ✓ Scale drawing

Critical Cross-Field Outcomes (CCFO)

Upon successful completion of this course, the learner will be able to:

- ✓ Identify and solve problems using critical and creative thinking: Solve a variety of problems involving space, shape and time using geometrical techniques related to the life or workplace of the learner
- ✓ Collect, analyse, organise and critically evaluate information: Gather, organise, and interpret information about objects and processes.
- ✓ Communicate effectively: Use everyday language and mathematical language to describe properties, processes and problem solving methods.
- ✓ Use mathematics: Use mathematics to analyse, describe and represent realistic and abstract situations and to solve problems relevant to the adult, the workplace and the local community.

WORK WITH PHYSICAL QUANTITIES

Outcome

Measure, estimate, and calculate physical quantities in practical situations relevant to the adult in life or the workplace

Outcome Range

- ✓ Basic instruments to include those readily available such as rulers, measuring tapes, measuring cylinders or jugs, thermometers, spring or kitchen balances, watches and clocks.
- ✓ In situations which necessitate it such as in the workplace, the use of more accurate instruments such as vernier calipers, micrometer screws, stop watches and chemical balances.
- ✓ Quantities to estimate or measure to include length/distance, area, mass, time, speed and temperature.
- ✓ Estimate the area and volume of simple irregular shapes and objects.
- ✓ The quantities should range from the low or small to the high or large.
- ✓ Mass, volume temperature, distance, and speed values are used in practical situations relevant to the learner or the workplace.
- ✓ Calculations involving the effects on area and volume when altering linear dimensions.
- ✓ Calculate heights and distances using Pythagoras` theorem.
- ✓ Calculate surface areas and volumes of right prisms (i.e., end faces are polygons and the remaining faces are rectangles) and cylinders from measurements in practical situations relevant to the life of the learner or in the workplace.

Assessment criteria

- ✓ Scales on the measuring instruments are read correctly.
- ✓ Quantities are estimated to a tolerance justified in the context of the need.
- ✓ The appropriate instrument is chosen to measure a particular quantity.
- ✓ Quantities are measured correctly to within the least step of the instrument.
- ✓ Calculations are carried out correctly.
- ✓ Symbols and units are used in accordance with SI conventions and as appropriate to the situation.

SI Units

The SI or Systéme International consists of 7 base units which were taken into use in order to have a worldwide acknowledged unit system. This has simplified the sharing of information between countries with different traditional units significantly.

QUANTITY	UNIT	SYMBOL
Mass	Kilogram	kg
Length	Meter	m
Time	Second	s
Temperature	Kelvin	K
Current	Ampére	A
Light	Candela	cd
Chemical standard unit	Mole	mol

It is VERY important to always indicate a unit. The unit is what gives meaning to a number. Just think 3000 tells you nothing about what this number is for or does, but R3000 is very useful! Also remember to indicate the unit EXACTLY as it is shown above. Km is wrong and so is S, if the unit is not given exactly right your answer will be wrong!

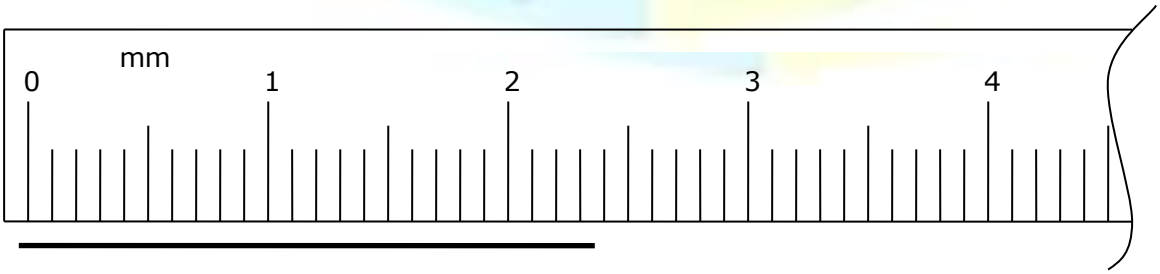
Length and Distance

We measure lengths in millimetres (mm), centimetres (cm), meters (m) and kilometres (km). These are the units of length in the SI (System International) Metric System.

The relations are: 1m = 100 cm = 1000mm and 1km = 1000m

The ***distance*** between two points is the path length between the two points.

Ruler

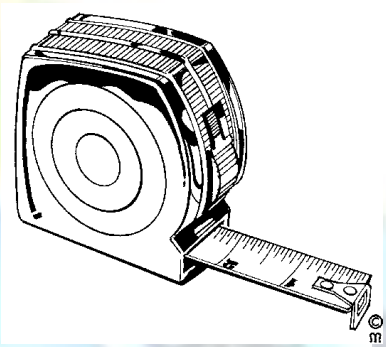


A ruler is a straight rigid strip of plastic, wood, metal, marked at regular intervals and used to draw straight lines or measure distances.

Each smallest increment (an increase in a number) represents 1 millimetre. Each 10th increment is marked with the relevant value. To measure the length of any straight line, place the ruler along that line so that one end of the line is at the zero mark. The other end will be at the number indicating its length. (24mm in this case)

Measuring Tape

A measuring tape will have similar markings and applications as a ruler. The main difference is that a measuring tape is designed for use over longer lengths. As a result increments of 100 mm and 1000 mm are also distinguished.



The length of a measuring tape usually starts at 1metre (1000 millimetre) and some can be as long as 100 metres. The measuring tape used by dress makers is usually 1metre or 1.5 metres long and the very long measuring tapes are used by people in the construction business.

Inside Caliper

Inside calipers are used to measure the internal size or internal cavity of an object.

- ✓ To use the upper caliper in the image you have to make manual adjustments before fitting or measuring. To finely set the caliper, you have to tap the caliper legs lightly on a handy surface until they will *almost* pass over the object. A light push against the resistance of the central pivot screw then spreads the legs to the correct dimension and provides the required, consistent *feel* that ensures a repeatable measurement.
- ✓ The lower caliper in the image has an adjusting screw that makes it possible to carefully adjust the tool without removing it from the work piece.
- ✓ Make sure that you do not accidentally adjust the vernier when moving it between the measured object and ruler
- ✓ The advantage of using a caliper is that its measurement is more accurate than many other measuring instruments.

Outside Caliper

Outside calipers are used to measure the external size of an object (the outer diameter of an object.)

The same observations and technique apply to this type of caliper, as for the inside caliper. Calipers can provide a high degree of accuracy and repeatability. They are especially useful when measuring over very large distances, for example when measuring a large diameter pipe. A vernier caliper (discussed next) does not have the depth capacity to straddle this large diameter while at the same time reach the outermost points of the pipe's diameter.

THREE OUTSIDE CALIPERS.

Vernier caliper

A variation to the more traditional caliper is the inclusion of a vernier scale, this makes it possible to directly obtain an accurate measurement.

Vernier calipers can measure internal dimensions (using the uppermost jaws in the picture at right), external dimensions using the pictured lower jaws, and depending on the manufacturer, depth measurements by the use of a probe that is attached to the movable head and slides along the centre of the body. This probe is slender and can get into deep grooves that may prove difficult for other measuring tools.

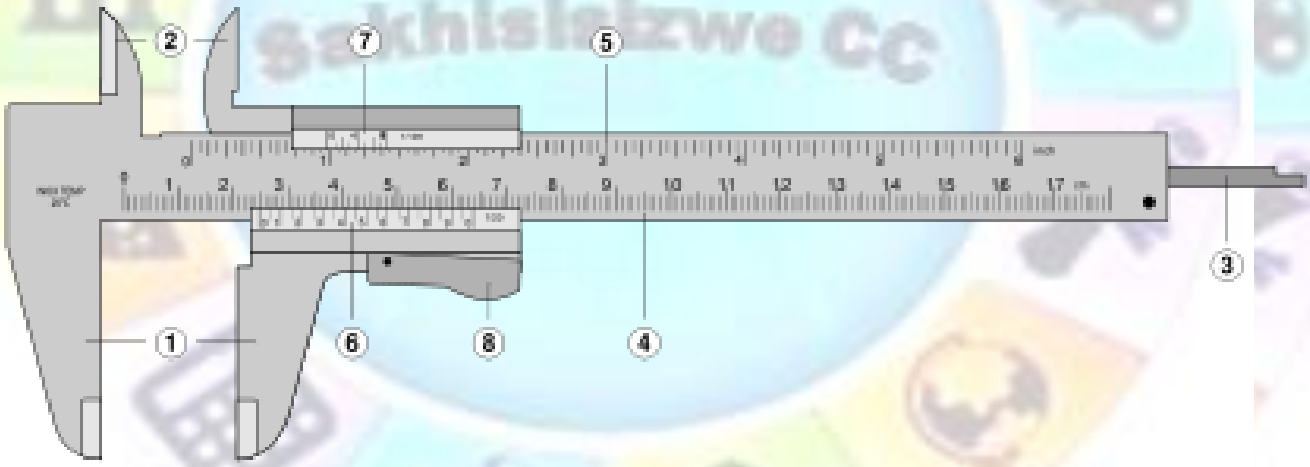
The vernier scales will often include both metric and Imperial measurements on the upper and lower part of the scale.

Vernier calipers commonly used in industry provide a precision to a hundredth of a millimetre (10 micrometres), or one thousandths of an inch.

A more accurate instrument used for the same purpose is the micrometer.

Parts Of A Vernier Caliper:

1. ***Outside jaws:*** used to measure external lengths
2. ***Inside jaws:*** used to measure internal lengths
3. ***Depth probe:*** used to measure depths
4. ***Main scale*** (cm)
5. ***Main scale*** (inch)
6. ***Vernier*** (cm)
7. ***Vernier (inch)***
8. ***Retainer: used to block movable part to allow the easy transferring of a measurement.***



Using The Vernier Caliper

A caliper must be properly applied against the part in order to take the desired measurement. For example, when measuring the thickness of a plate a vernier caliper must be held at right angles to the piece. Some practice may be needed to measure round or irregular objects correctly.

Accuracy of measurement when using a caliper is highly dependent on the skill of the operator. Regardless of type, a calliper's jaws must be forced into contact with the part being measured. As both part and caliper are always to some extent elastic, the amount of force used affects the indication. A consistent, firm touch is correct. Too much force results in an under indication as

part and tool distort; too little force gives insufficient contact and an over indication. This is a greater problem with a caliper incorporating a screw, which lends mechanical advantage.

Simple calipers are uncalibrated; the measurement taken must be compared against a scale. Whether the scale is part of the caliper or not, all analogue calipers -- verniers and dials -- require good eyesight in order to achieve the highest precision. Digital calipers have the advantage in this area.

Calibrated calipers may be mishandled, leading to loss of zero. When a caliper's jaws are fully closed, it should of course indicate zero. If it does not, it must be recalibrated or discarded. It might seem that a vernier caliper cannot get out of calibration but a drop or knock can be enough. Sometimes a careful tap is enough to restore zero. Digital calipers have zero set buttons.

Micrometer Screws

Micrometer is a name generally given to any device for measuring small angles or dimensions, usually smaller than 1mm.

A micrometer screw displaces the pointer uniformly by turning a screw. If, for example, the step of the screw is 0.5mm and the screw head is read to 1/1000 of a revolution, we measure to 0.0005mm, which is about equal to the wave length of light.

History

The first ever micrometric screw was invented by William Gascoigne in the 17th century, as an enhancement of the Vernier; it was used in a telescope to measure angular distances between stars. Its adaptation for the measurement of the small dimension was made by Jean-Louis Palmer; this device is therefore often called palmer in France. In 1888 Edward Williams Morley added to the precision of micrometric measurements and proved their accuracy in a complex series of experiments.

Micrometer (Device)

EXTERNAL, DEPTH *INTERNAL, AND MICROMETERS*

A micrometer is a widely used device in mechanical engineering for precisely measuring thickness of blocks, outer and inner diameters of shafts and depths of slots. Appearing frequently in metrology, the study of measurement, micrometers have several advantages over other types of measuring instruments like the Vernier caliper.

Types

The image shows three common types of micrometers, the names are based on their application:

An external micrometer is typically used to measure wires, spheres, shafts and blocks. An internal micrometer is used to measure the opening of holes, and a depth micrometer typically measures depths of slots and steps.

The precision of a micrometer is achieved by a using a fine pitch screw mechanism.

An additional interesting feature of micrometers is the inclusion of a spring-loaded twisting handle. Normally, one could use the mechanical advantage of the screw to force the micrometer to squeeze the material, giving an inaccurate measurement. However, by attaching a handle that will ratchet at a certain torque, the micrometer will not continue to advance once sufficient resistance is encountered.

Reading A Metric Micrometer

MICROMETER THIMBLE READING 5.78MM

The spindle of an ordinary metric micrometer has 2 threads per millimetre, and thus one complete revolution moves the spindle through a distance of 0.5 millimetre. The longitudinal line on the frame is graduated with 1 millimetre divisions and 0.5 millimetre subdivisions. The thimble has 50 graduations, each being 0.01 millimetre (one-hundredth of a millimetre). To read a metric micrometer, note the number of millimetre divisions visible on the scale of the

sleeve, and add the total to the particular division on the thimble which coincides with the axial line on the sleeve.

Suppose that the thimble were screwed out so that graduation 5, and one additional 0.5 subdivision were visible (as shown in the image), and that graduation 28 on the thimble coincided with the axial line on the sleeve. The reading then would be 5.00 +0.5 +0.28 = 5.78 mm.

Reading A Vernier Micrometer

MICROMETER SLEEVE (WITH VERNIER) READING 5.783MM

Some micrometers are provided with a vernier scale on the sleeve in addition to the regular graduations. These permit measurements within 0.001 millimetre to be made on metric micrometers, or 0.0001 inches on inch-system micrometers.

Metric micrometers of this type are read as follows: First determine the number of whole millimetres (if any) and the number of hundredths of a millimetre, as with an ordinary micrometer, and then find a line on the sleeve vernier scale which exactly coincides with one on the thimble. The number of this coinciding vernier line represents the number of thousandths of a millimetre to be added to the reading already obtained.

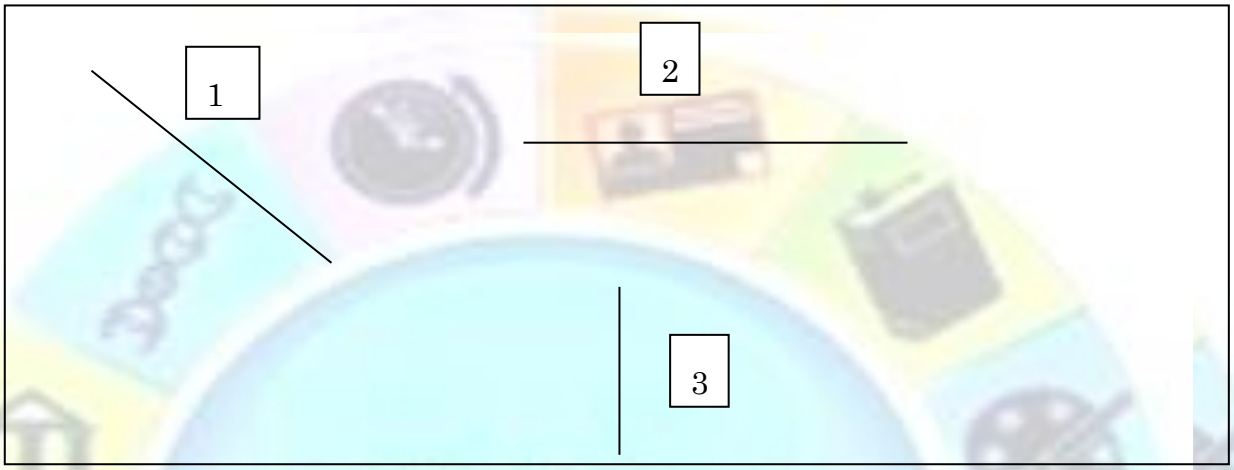
Thus, for example, a measurement of 5.783 millimetres would be obtained by reading 5.5 millimetres on the sleeve, and then adding 0.28 millimetre as determined by the thimble. The vernier would then be used to read the 0.003 (as shown in the image).

Inch micrometers are read in a similar fashion.

Note: 0.01 millimetre = 0.000393 inch, and 0.002 millimetre = 0.000078 inch (78 millionths) or alternately, 0.0001 inch = 0.00254 millimetres. Therefore, metric micrometers provide smaller measuring increments than comparable inch unit micrometers—the smallest graduation of an ordinary inch reading micrometer is 0.001 inch; the vernier type has graduations down to 0.0001 inch (0.00254 mm). When using either a metric or inch micrometer, without a vernier, smaller readings than those graduated may of course be obtained by visual interpolation between graduations.

Formative Assessment

Estimate (guess) and then measure the lengths of the following line segments. Give your answers in cm and mm.



A rectangular plot of land has a length of 250m and a width of 175 m. The farmer wants to fence the plot using 6 strands of wire. What length of wire will s/he need? Show your calculations.

Speed

Speed = $\frac{distance}{time}$

Example

You take 5 minutes to walk the 900 m from your house to the mall. Your average speed was

Speed = $\frac{900m}{300s}$ = 3 m/s

Note that we do not know your speed at any specific point on your journey. To know that, we need to know what distance was covered in a very short time period around that point.

Important: Speed is measured in m/s therefore minutes must be converted to seconds.

Formative Assessment

Calculate:

Every weekday, from Monday to Thursday, Thabo drives the distance of 25 km between his home and work in 30 minutes. However, Fridays it only takes him 20 minutes to cover the same distance.

Calculate his speed on weekdays except Fridays.
Calculate his speed on Fridays.

A bicycle moves 65 meters in 15 seconds. Calculate the speed of the bicycle.

Area

The amount of surface covered by a flat figure is called the area of the figure. We measure area by counting the number of unit squares that cover the figure. A unit square with sides of 1 cm each has an area of $1\ cm^2$.

Formulae to use in calculations are supplied in Table 1.

1. Decide which shape it is to choose the relevant formula.
2. Write down the appropriate formula
3. Write down what you want to calculate and information given to you.
4. Substitute in appropriate formula
5. Calculate your answer (Remember to use the correct unit)

Example:

a). Calculate the area of the figure:

Decide which formula –Area of Rectangle:

A = lw (Area = length x width)

A = ?

l = 5m

w = 3m

A = 5m x 3m

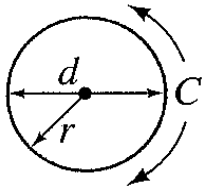
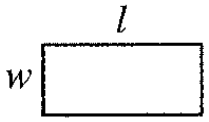
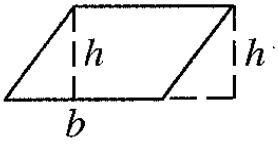
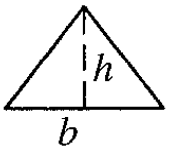
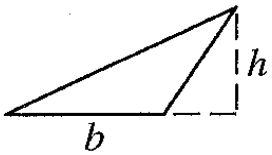
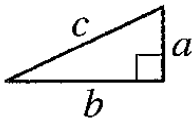
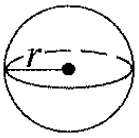
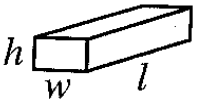
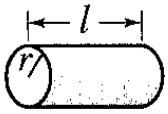
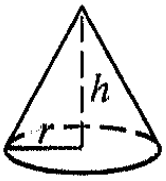
A = $15m^2$

Compiled by: Sakhisisizwe	Approved By: L.Levin

Formulae

Table 1:

Useful Geometry Formulas—Areas, Volumes

Circumference of circle	$C = \pi d = 2\pi r$	
Area of circle	$A = \pi r^2 = \dfrac{\pi d^2}{4}$	
Area of rectangle	$A = lw$	
Area of parallelogram	$A = bh$	
Area of triangle	$A = \frac{1}{2} hb$	 
Right triangle (Pythagoras)	$c^2 = a^2 + b^2$	
Sphere: surface area volume	$A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	
Rectangular solid: volume	$V = lwh$	
Cylinder (right): surface area volume	$A = 2\pi rl + 2\pi r^2$ $V = \pi r^2 l$	
Right circular cone: surface area volume	$A = \pi r^2 + \pi r\sqrt{r^2 + h^2}$ $V = \frac{1}{3}\pi r^2 h$	

b).Calculate the area of the figure:

Decide which shape –Area of Triangle:

A = $\frac{1}{2}bh$ (From Table 1)

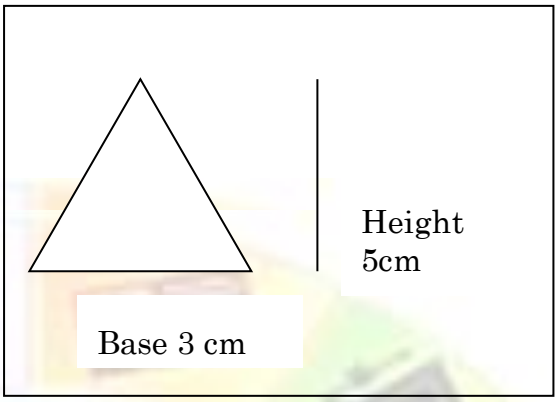
A = ?

b = 3cm

h = 5cm

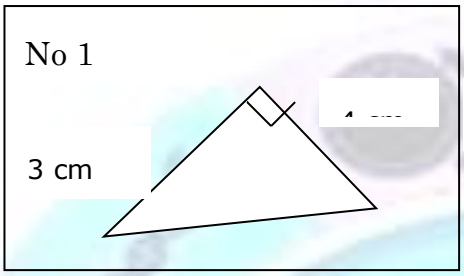
A = $\frac{1}{2}$ x 3cm x 5cm

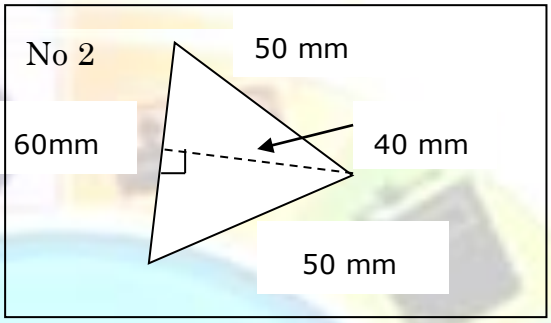
A = 7.5cm^2

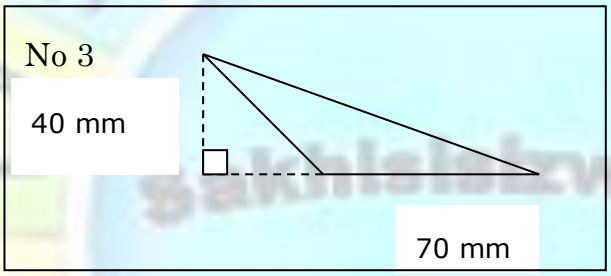


Formative Assessment

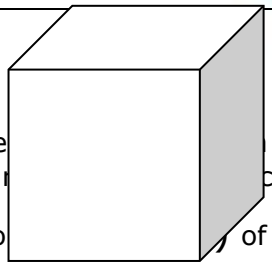
Estimate and then calculate the area of:
This page
The top of your file
Which units of area will be best for measuring
the area of the floor of this room?
the area of a soccer field?
the area of South Africa?
Calculate the area of each of the following triangles







Consider a box with dimensions: length 34 cm, width 4 cm and height 3 cm. How many 1 cm3 unit cubes are needed to cover the base of the box? How many layers of cubes are needed to fill the box?



Volume

A cube each is a unit cube with volume $1\ cm^3$ (one cubic centimetre). We measu container or solid in terms of the number of unit cubes needed to fill it.

The vo of any rectangular box with length l (in cm), width w (in cm) and height h (in cm) is: $V = l \times w \times h$

For very small volumes, we can use unit cubes of $1\ mm^3$ (one cubic millimetre). For big volumes, we can use unit cubes of $1\ m^3$ (one cubic metre). The relationship between these cubes is:

1 cm^3 = 1000 mm3

1 m3 = 1000000 cm3

Formative Assessment

Estimate (guess) which object has the greater volume: a cube with sides of 6 cm or a rectangular box of 7 cm by 6 cm by 4 cm. Now calculate the volumes accurately.

Measuring Fluids

In the metric system, the units used to measure capacity are the litre and millilitre. When a solid is dropped into water, the object takes the place of some of the water. We see that the level of the water rises. One millilitre (1 ml) of water is the volume of water that is displaced by 1 cm^3. Or we can say that 1 ml of water fills 1 cm^3.

Fluids such as water, milk and cold drinks are measured in millilitres or litres.

One litre = 1 000 ml.

For big volumes of fluid we can use the kilolitre (kl) as unit. 1 kl = 1 000 l.

Example: 5 ml of fluid fills $5\ cm^3$

¼ l = 250 ml

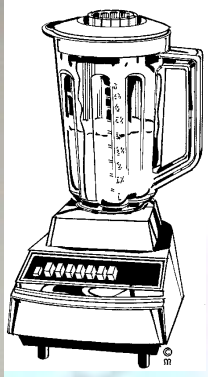
1 kl of fluid fills 1 000 000 cm^3 or $1\ m^3$

Formative Assessment

Give some examples of fluids that you can buy in packages that are marked in

ml
l

Measuring Cylinders



In this case each small increment represents 10 millilitre. Every 100 ml has its value indicated.

To acquire a certain amount of a liquid or powdery solid it is poured into the measuring cylinder. The marking next to the flat level of the substance would indicate the volume contained.

Measuring cylinders are used every day by people baking cakes, cooking, as well as by hairdressers laboratory technicians, pharmacists, students studying chemical science, chemical scientists and at times even barmen.

Measuring cylinders are used to measure the amount of water or liquid and/or powdery solid in order to:

- ✓ Mix hair colouring
- ✓ Mix batter for cake, where you would add milk or water to flour, salt, sugar and other powdery solids
- ✓ Mix the amounts of alcoholic beverages to make a cocktail or other drink
- Mix chemical substances which can be in liquid or powder form.

Mass

What is the difference between weight and mass?

We say that the weight ("heaviness") of an object depends on its mass. The bigger the mass, the bigger the pull of the earth is on it.

To measure mass we choose a unit of mass and express the mass of an object in this unit. In the metric system we use the gram (g) and the kilogram (kg) as units of mass. 1kg = 1000g, 1g = 1000 mg

Remember to use the same units when comparing the masses of different objects.

Spring Balance

A balance is an instrument for comparing the weights of two bodies to determine the difference in mass.

A spring balance is a balance that measures weight by the tension of a spring, in other words you hang the object you want to weigh from the spring balance. Fishermen use this to weigh the fish they have caught in competitions. Butchers also use spring balances to weigh carcasses.

Hang a spring balance like this from any support strong enough for the object to be weighed. Attach the bottom hook to the object. The indicator shows the mass of the object.

A spring scale (or spring balance) is a weighing scale often used to measure force, such as the force of gravity, exerted on a mass or the force of a person's grip or the force exerted by a towing vehicle. This force is commonly measured in newtons.

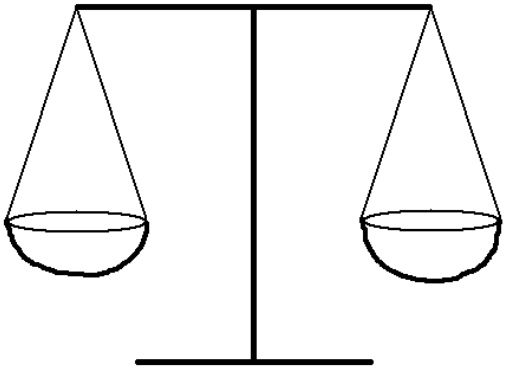
Many spring scales are marked right on their face "Not Legal for Trade" or words of similar import. Some spring scales can be calibrated for the accurate measurement of mass in the location in which they are used. The spring scale works on Hooke's Law.

If the two spring scales are hung one below the other both will read the weight of the body hung on the lower scale.

Spring scales come in different sizes. Generally, small scales that measures newtons will have a weaker spring than larger ones that measures 10's, 100's or 1000's of newtons

Chemical balances

We have said that a balance is an instrument for comparing the weights of two bodies to determine the difference in mass. In the old days a balance consisted of a machine where weights were added to a pan on one side and the goods that had to be weighed were placed in a pan on the other side until the pans were at the same height.



So, if you want to measure 500g flour, you would put 500g weights in one pan, which would cause that pan to be heavier and sink to the bottom. You would then add flour in the other pan until both pans are level, then you should have 500g worth of flour.

A chemical balance is a very sensitive balance designed to measure very small weights accurately. A pharmacist or laboratory technician would have to measure powder and other substances weighing as little as 5grams or 1 gram and they would use a chemical balance.

Laboratory Balances

METTLER DIGITAL ANALYTICAL BALANCE WITH 0.1 MG PRECISION.

An analytical balance is an instrument used to measure mass to a very high degree of precision. The weighing pan(s) of a high accuracy (0.1 mg or better) analytical balance are inside a see-through enclosure with doors so dust does not collect and so any air currents in the room do not affect the delicate balance. Also, the sample must be at room temperature to prevent natural convection from forming air currents inside the enclosure, affecting the weighing.

Very precise measurements are achieved by ensuring that the fulcrum of the beam is friction-free (a knife edge is the traditional solution), by attaching a pointer to the beam which amplifies any deviation from a balance position; and finally by using the lever principle, which allows fractional weights to be applied by movement of a small weight along the measuring arm of the beam.

Sources Of Error

Some of the sources of potential error in a high-precision balance include the following:

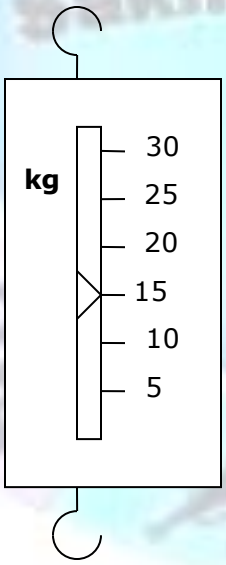
- ✓ Buoyancy, due to the fact that the object being weighed displaces a certain amount of air, which must be accounted for. High-precision balances are often operated in a vacuum.
- ✓ Air gusts, even small ones, may push the scale up or down.
- ✓ Friction in the moving components may prevent the scale from reaching equilibrium.
- ✓ Settling airborne dust may contribute to the weight.
- ✓ Scale may be mis-calibrated.
- ✓ Mechanical components may be mis-aligned.
- ✓ Magnetic fields from nearby electrical wiring may act on iron components.
- ✓ Magnetic disturbances to electronic pick-up coils or other sensors.
- ✓ Forces from electrostatic fields, for example, from feet shuffled on carpets on a dry day.
- ✓ Chemical reactivity between air and the substance being weighed (or the balance itself, in the form of corrosion).
- ✓ Condensation of atmospheric water on cold items.
- ✓ Evaporation of water from wet items.
- ✓ Convection of air from hot or cold items.
- ✓ The Coriolis force from Earth's rotation.
- ✓ Vibration and seismic disturbances; for example, the rumbling from a passing truck.

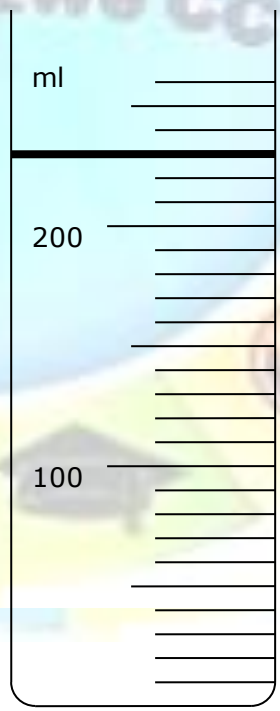
Symbology

The weighing scales (specifically, a beam balance) are one of the traditional symbols of justice, as wielded by statues of Lady Justice. This corresponds to the use in metaphor of matters being "weighed up" or "held in the balance".

Formative Assessment

In each case give the greater/greatest measurement:
250 g; 0.2 kg
0.01 kg; 12 000 mg; 10 g
What is the mass indicated on the spring balance shown?
What is the volume of the fluid in the measuring cylinder shown age?





Temperature

Temperature scales

There are three commonly used temperature scales:

- The Celsius scale is the most commonly used temperature scale.
- The Fahrenheit scale is used in the United States.
- The absolute or Kelvin scale is used in scientific work.

The Fahrenheit and Celsius scales assign arbitrary values to both freezing and boiling points of water at atmospheric pressure.

	Celsius	Fahrenheit
Freezing point	0.00°C	32.0°F
Boiling point	100°C	212°F

Between these two reference points the Celsius scale is divided into 100 equal units and the Fahrenheit scale into 180 equal units. This makes it easy to convert from Celsius to Fahrenheit or vice versa as each value of Celsius has a corresponding Fahrenheit value, 1°F = 5/9°C. The conversion formulas are as follows:

T(°C) = 9/5[T(°F)-32] or T(°F) = 9/5T(°C) + 32

It may be easier to simply remember that 0°C=32°F and that 5°C=9°F.

Example:

Taking your temperature

Normal body temperature is 98.6°F. What is this in °C? And what is the temperature in Fahrenheit back from °C?

Solution:

Convert from Fahrenheit to Celsius using the formulae:

T(°C) = 9/5[T(°F)-32]

°F = 98.6

°C = ?

Substitute in formulae above

T(°C) = 5/9 x [98.6 – 32]

T(°C) = 5/9 x [66.6]

T(°C) = 37°C

Convert Celsius into Fahrenheit using the formula:

T(°C) = 9/5[T(°F)-32]

°C = 37

°F = ?

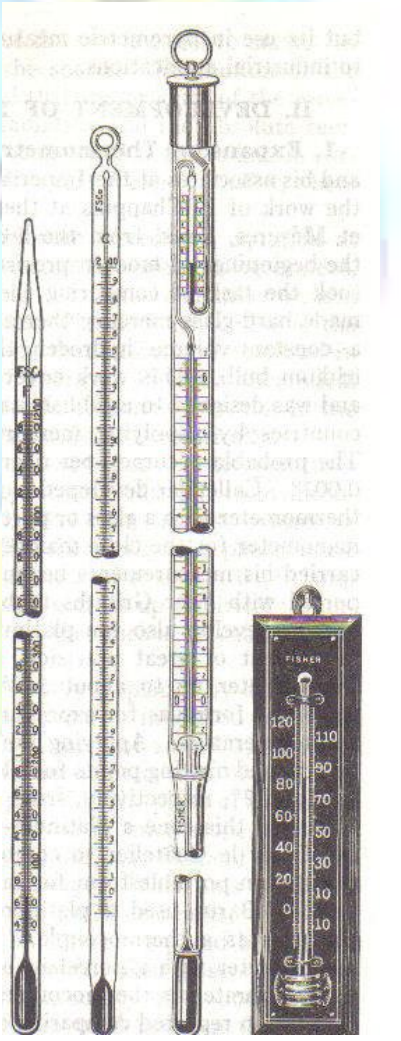
Substitute in formulae given above

T(°F) = 9/5 x 37°C + 32

T(°F)= 66.6°C + 32

T(°F) =98.6 °F

It is important to remember that different thermometers are made from various materials and filled with different substances, in practice this means that they all expand and contract differently in response to changes in temperature. Because of this most thermometers are only reliable within a set range of temperatures.



Change Of Temperature

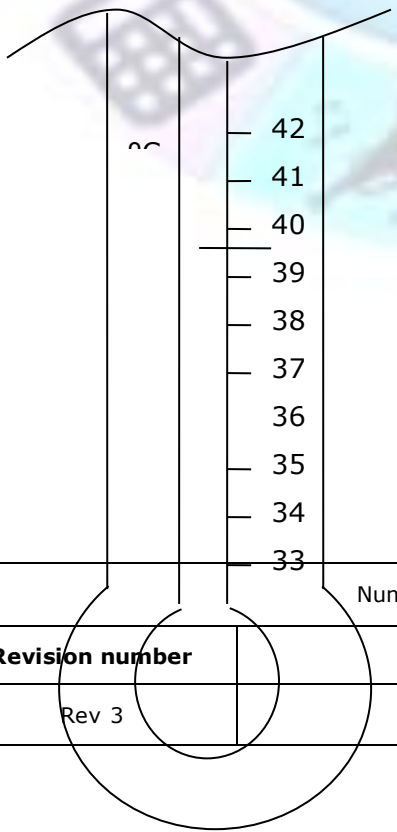
Changes of temperature in time can be calculated as follows:

Δt (change in temperature) = hf (heating factor ºC/s) × time

Note that this formula expresses any type of linear quantity change with respect to time, and can be used in various applications.

Instruments used

Thermometer



The example left looks like a typical thermometer used to measure body temperature. It contains mercury or a coloured liquid where the level indicates the ambient temperature.

A thermometer is an instrument for measuring or sensing temperature, typically consisting of a graduated glass tube containing mercury or alcohol which expands when heated.

Thermometers are used by doctors, nurses and medical staff to determine the temperature of a patient. A patient with a higher than normal temperature, **36ºC**, would indicate illness, as 36 degrees Celsius is the normal body temperature for human beings.

Thermometers are also used by the weather bureau to determine the daily temperatures. You can also buy a thermometer to determine the temperature in your house on a day to day basis and swimming pool owners use them to find out what the water temperature is.

Some thermometers used by medical staff and found in households are shown on the right. You may have seen one or more of them during visits to the doctor or hospital.

Thermometers that make use of digital display have temperature influenced components that generate code. This code is processed and the relevant temperature is displayed as follows: **36ºC**

In some countries, such as the USA, temperature is measured in Fahrenheit, but in South Africa temperature is measured in Celsius.

In Celsius, **0ºC** is the point at which water freezes and **100ºC** is the point at which water boils. Of course, the freezing and boiling point of water as indicated above is at sea level, the exact temperature changes a little bit as you move farther inland and higher than sea level.

Formative Assessment

A bucket of water has to be heated from 20 ºC to 45 ºC. The heating element can heat at 0.05 ºC/s (heating factor). How many seconds would it take this element to heat the water to the required temperature?
The inlet of a boiler adds 6 Litres of water per second to its contents. We will call this a flow factor (ff). The volume needs to increase from 1000 Litres to 1500 Litres. How many seconds would this take?
First calculate the change in volume.

Next use the flow factor(ff) of 6 Liters per second to calculate the time required.
What is the difference in temperature in degree Celsius between Ice and boiling water:
The maximum and minimum temperatures for a day (estimate!) in June:

Time

Clocks And Wristwatches

A clock is an instrument that measures and indicates the time. A watch is a small timepiece usually worn on a strap on one's wrist. So we use watches and clocks to tell the time.

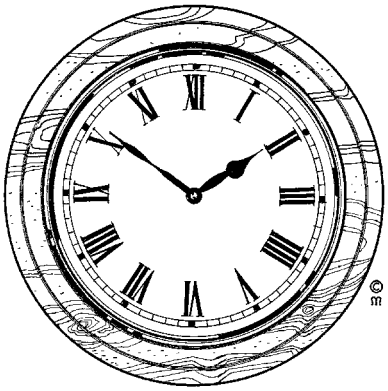
Clocks like these indicate the minutes between hours with the long arm and the hours with the short one. The numbers are indicated in Roman Numerals.

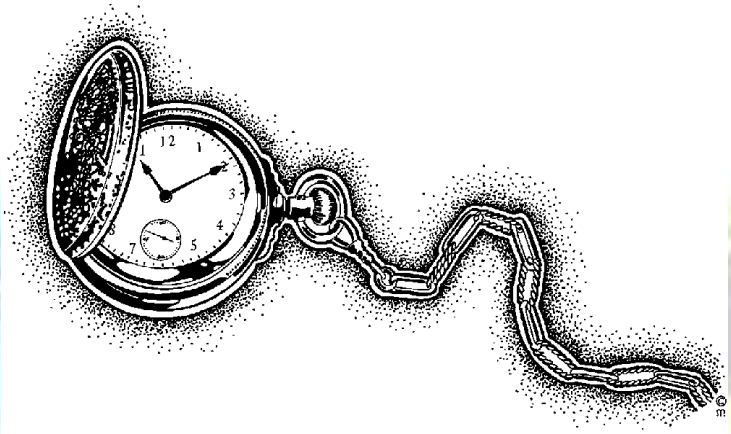
Every hour marking indicates the hour to be read with the short arm. It also indicates 5 minute increments to be read with the long arm.

The minute indication starts with 5 minutes past the hour at 1, and ends with 55 minutes past (5 minutes before) at 11.

Before wristwatches were common, most people, churches and government buildings used clocks to tell the time. These days clocks are not commonly found, except in church towers and government buildings. Most of us use watches to tell the time.

Luckily, watches are no longer commonly numbered in Roman numerals, but rather the numbers as we use them from day to day. This watch only indicates hours (12), half hours (6) and quarter hours (3) and (9). It is left up to the wearer of the watch to work out when it is 5 past 10 or 20 to 7.





Units Of Time

The basic unit of time is the second (s). We can also measure time in minutes (min), hours (h), days, weeks and so on. There are 7 days in a week, 24 hours in a day, 60 minutes in an hour and 60 seconds in a minute.

The face of a watch with hands is divided into 12 divisions. The hours between 12 o'clock midday and 12 o'clock midnight used to be written as 1 p.m, 2 p.m etc up to 12 p.m (midnight). The hours after midnight used to be written as 1 a.m., 2 a.m. etc up to 12 a.m (midday).

Digital Time

Today we use the international system of time. In this system the hours after midnight are counted 01:00, 02:00 and so on. Midday is 12:00 and midnight is 24:00. The digits before the ":" show the hours and the digits after the ":" show the minutes. Digital watches show time in this way.

16:30:00

Digital watches do the same thing as ordinary wristwatches, the only difference is that they show the time differently. The time on your cell phone or PC screen is shown digitally:

The digits display the current time. AM is for morning and PM is for afternoon. The 16 indicates the current hour, which is four o' clock. The 30 indicates the minutes and the 00 the seconds. The time on this digital watch is 30 minutes past four o' clock.

The time as shown on a PC screen.

Stopwatches

A stopwatch takes the time of an event chosen by the user. An example of this would be the time between the beginning and end of a race. Most stopwatches can measure split seconds as small as 1/1000 of a second. The '00' on the left indicates minutes. The figure in the middle indicates the seconds and the figure on the right indicates the split seconds.

Formative Assessment

How many seconds are there in 2 minutes?
How many minutes are there in 3 h 45 min?
How many seconds are there in 610,2 minutes?
Write the following according to the international time system:
2.16 p.m.

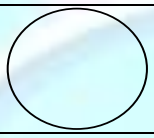
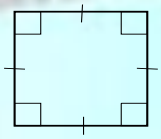
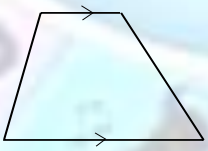
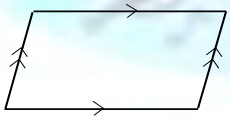
12.05 p.m.
3.12 a.m.
An athlete runs 1 500 m in 3 min. 42 s. How many seconds is this less than 4 min.
How many seconds is this more than 3 min 38 s.

Two-dimensions (2D) and areas

The purpose of this section is to introduce you to two-dimensional objects in terms of their various shapes in order to determine their areas and symmetries.

Areas are always in demand for many different uses. This section shows you how to calculate some of these so that you can estimate the surface areas when you need them.

Below is a summary various 2D shapes. The name, a small drawing and a short description of each shape is shown in order to provide you with an overview of what follows.

Name	***Drawing***	***Description***
Circle		The edge of the circle is at a constant distance from the middle. This distance is called the radius.
Triangle		A triangle has three straight sides.
Square		A square has four equal sides and four right angles.
Rectangle		A rectangle has the opposite sides of equal length and four right angles.
Trapezium		A trapezium has one parallel pair of opposite sides.
Parallelogram		A parallelogram has both opposite sides equal and parallel.

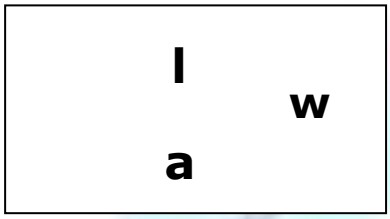
VARIOUS 2D SHAPES

Note that small lines drawn through the edges of an item indicate that those edges (lines) have the same length. The parallelogram is an example that shows two pairs of equal lines. A small square in a corner indicates a right angle of 90 degrees (90°). The square is an example that has four right angles. The greater than signs **(>)** indicate lines that are parallel to one another. The parallelogram has two parallel sides.

There are many geometric formulas, relating height, width, length, or radius to perimeter, area, surface area volume. Some of the formulas are rather complicated, and you have hardly seen them, let alone used them. But there are some basic formulas you have to remember.

The area and perimeter of a rectangle

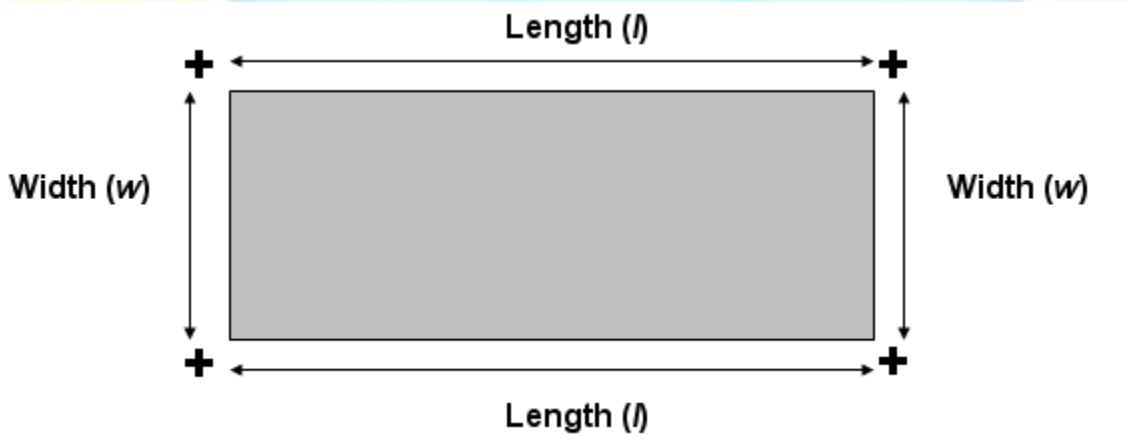
A plane figure with four straight sides and four right angles and with unequal adjacent sides.



Area(a) = l × w (unit: m ²)

The Perimeter of a Rectangle

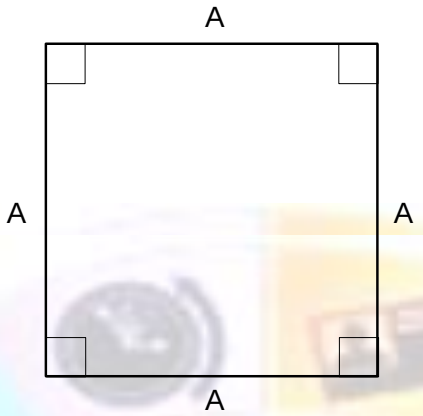
If you look at the picture of a rectangle, and remember that "perimeter" means "length around the outside", you'll see the rectangle's perimeter is the sum of the top and bottom lengths (*l*) and the left and right widths (*w*):



$$P_{rect} = 2l + 2w$$

The Area and Perimeter of a Square

A square is a four-sided figure in which all four sides are the same length, they are parallel to one another and the angle between each adjacent side is at right angles to its neighbour. It's a lot easier to see a square than to describe one.



A SQUARE SHOWING ALL SIDES ARE EQUAL, PARALLEL AND AT RIGHT ANGLES TO ONE ANOTHER

The sides all have the same length, A, and each side is parallel to the opposite side and at 90 degrees to its neighbours. The square in each corner indicates that these are right angles.

Area(a) = l × w (unit: m ²)

If the side of a square is 12 centimetres, what is its area? The area is 12 × 12 = 144 so its area is 144 square centimetres (cm^2).

Squares are therefore simpler, because their lengths and widths are identical. The area and perimeter of a square versus length (*s*) are given by:

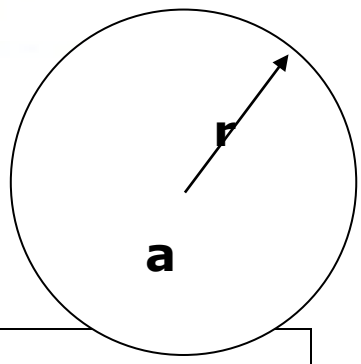
$$P_{sqr} = 4s$$

Circle

A round plane figure whose boundary is made up of points at an equal distance from the centre. The area of a circle is a bit more complicated to calculate but not difficult. Below is a circle with a radius, r.

The radius is the measurement from the centre of the circle to its boundary. Note that the radius is always the same in the same circle no matter the angle it is drawn at. The diameter is the cross section of the circle and is always twice the length of the radius.

An irrational number, π (Greek letter 'pi'), is used in circular calculations. An irrational number is one that has an infinite number of digits after the decimal point. In addition, the decimal portion of an irrational number does not have a pattern of digits that repeat and never ends in zero. Furthermore, irrational numbers cannot be represented by a fraction.



Area (a) = π × r ² (unit: m ²)

The area of the circle is $\pi \times r^2$, where π = 3.14159265, or simply 3.14, approximately.

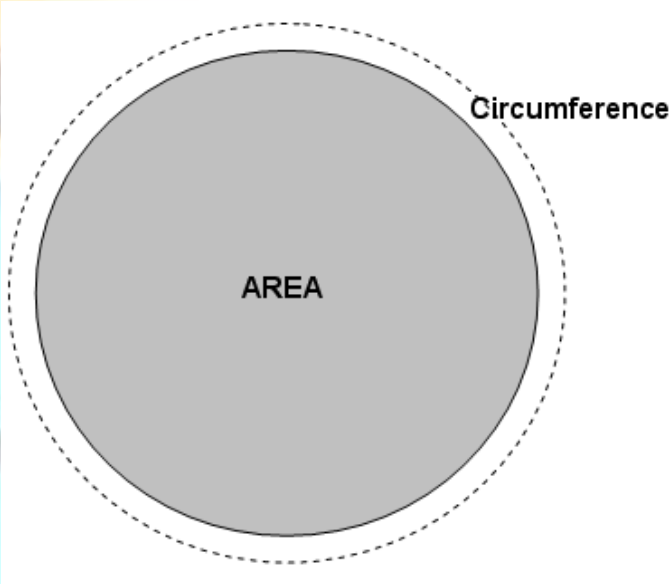
So the area of a circle is **$3.14 \times r^2$.**

For an example, if the radius of a circle is 8 metres, the area would be $3.14 \times 8^2 = 3.14 \times 64 = 200.96$ square metres (m^2), approximately.

Many people use $\frac{22}{7}$ as an approximation for **π.**

$\frac{22}{7}$ = 3.1429 rounded to 4 decimal places (ten-thousandth)

You should know the formula for the circumference *C* and the area *A* of a circle, or given the radius *r:*



A_{cir} = (pi)r^2

Circumference: *C_{cir} = 2(pi)r*

("pi" is the number approximated by 3.14159)

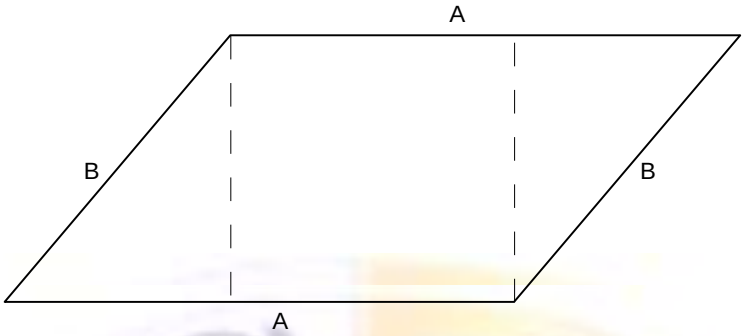
The circumference of the circle is $2 \times \pi \times r$, where $\pi = 3.14$ and $2 \times \pi = 6.28$, approximately. So the circumference of a circle is $6.28 \times r$.

For an example, if the radius of a circle is 8 metres, the circumference would be $6.28 \times 8 = 6.26 \times 8 = 50.08$ metres (m), approximately.

Remember that the radius is the distance from the centre to the outside of the circle. In other words, the radius is ***halfway*** across. If you deal with the diameter of a circle, the length of a line going all the way across, then you have to divide in half to apply the above formulas.

Parallelogram

A parallelogram is rectangle with a tilt. All sides are parallel but the angles between the sides differ.



In order to help you visualize a parallelogram, I drew in vertical lines to form a right-angled triangle from the intersection of the sides A and B to the opposite side. Notice what this figure is showing us. If I cut the left triangle off the parallelogram and stick it on the right side, I have a rectangle! Therefore, the parallelogram is nothing but a rectangle with a tilt. The tilt is called a 'shear' in many industries. (And it has nothing to do with sheep!) You won't find parallelograms with the dashed lines so don't expect to see them. However, you should be able to look at a parallelogram, or something close to one, by putting in the dashed lines mentally.

In order to calculate the area of a parallelogram I use exactly the same formula to calculate the area of a rectangle: Area = A × B.

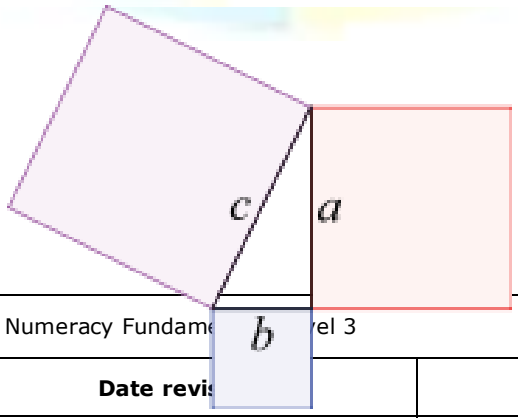
Observations for square, rectangle, parallelogram and rhombus

The rectangle, square and parallelogram have the same characteristics: each has two pairs of parallel sides. Therefore, each one is simply a variation on the parallelogram and all their areas are calculated as base × height. The square is a rectangle with its base and height equal. The rectangle is a parallelogram with straight sides and the rhombus is a parallelogram with an equal base and height.

The most general of these four figures is the parallelogram: it has two parallel sides. And nothing is said about the lengths of these sides or the angle between the two sets of parallel sides. A rectangle is a parallelogram with right angles (90 degrees). A square and a rhombus have equal sides.

Theorem Of Pythagoras

In mathematics, the Pythagorean theorem or Pythagoras' theorem is a relation in Euclidean geometry among the three sides of a right triangle. The theorem is named after the Greek mathematician Pythagoras, who by tradition is credited with its discovery, although knowledge of the theorem almost certainly predates him. The theorem is known in China as the "Gougu theorem" (勾股定理) for the (3, 4, 5) triangle.

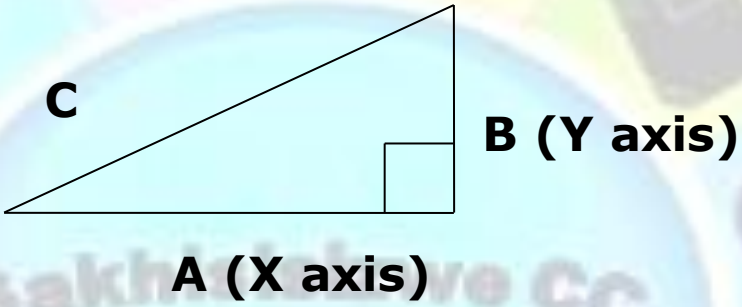


The Pythagorean theorem

The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

Probably one of the most important theorems of practical geometry it states that in a right triangle, ***the square of the length of the hypotenuse equals the sum of the lengths of the remaining two sides***. Looking at a right triangle on a x vs. y axis you would see that the following equation represents the relationship between the lengths of the sides.

$a^2 + b^2 = c^2$



To state the theorem in a different way::

In any right triangle, the area of the square whose side is the hypotenuse (the side of a right triangle opposite the right angle) is equal to the sum of areas of the squares whose sides are the two legs (i.e. the two sides other than the hypotenuse).

If we let c be the length of the hypotenuse and a and b be the lengths of the other two sides, the theorem can be expressed as the equation

$$a^2 + b^2 = c^2$$

or, solved for c:

$$\sqrt{a^2 + b^2} = c.$$

This equation provides a simple relation among the three sides of a right triangle so that if the lengths of any two sides are known, the length of the third side can be found. A generalization of this theorem is the law of cosines, which allows the computation of the length of the third side of any triangle, given the lengths of two sides and the size of the angle between them.

The converse of the theorem is also true:

For any three positive numbers a, b, and c such that $a^2 + b^2 = c^2$, there exists a triangle with sides a, b and c, and every such triangle has a right angle between the sides of lengths a and b.

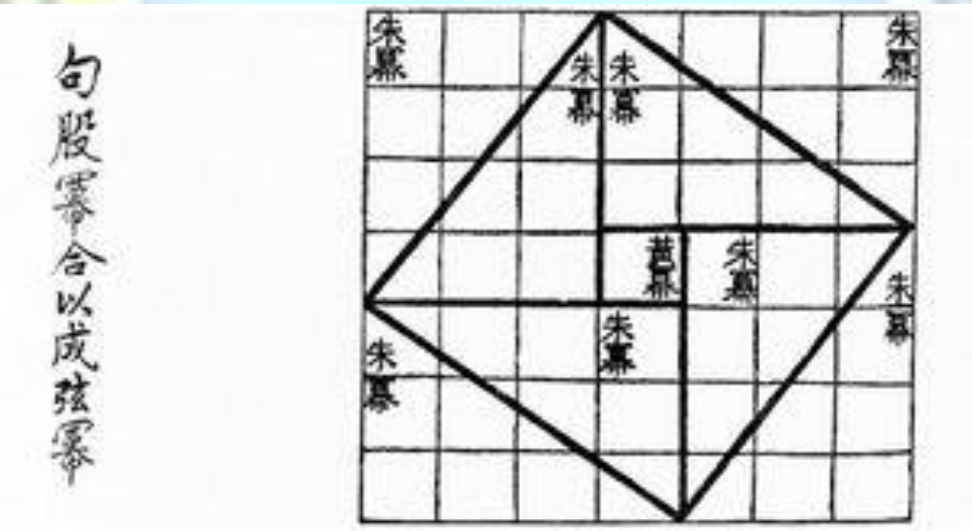
We can also use this theorem to determine whether a triangle is right, obtuse, or acute, as follows.

If $a^2 + b^2 = c^2$, then the triangle is right.

If $a^2 + b^2 > c^2$, then the triangle is acute.

If $a^2 + b^2 < c^2$, then the triangle is obtuse.

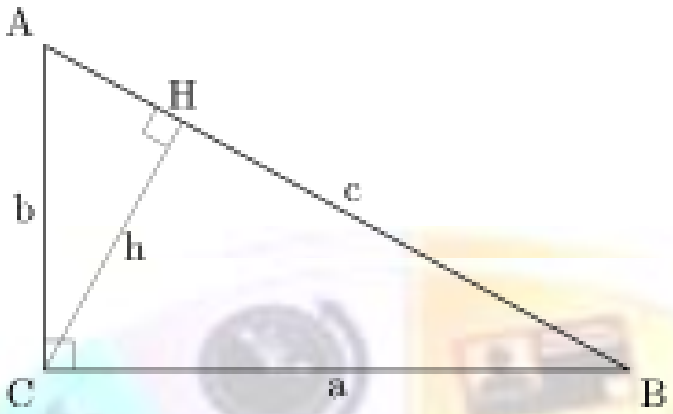
VISUAL PROOF FOR THE (3, 4, 5) TRIANGLE AS IN THE CHOU PEI SUAN CHING 500–200 BC



Proofs

This theorem may have more known proofs than any other (the law of quadratic reciprocity being also a contender for that distinction); the book *Pythagorean Proposition*, by Elisha Scott Loomis, contains 370 proofs. For the purposes of this training intervention we will discuss just one of them.

Proof Using Similar Triangles



ike many of the proofs of the Pythagorean theorem, this one is based on the proportionality of the sides of two similar triangles.

Let ABC represent a right triangle, with the right angle located at C, as shown on the figure. We draw the altitude from point C, and call H its intersection with the side AB. The new triangle ACH is similar to our triangle ABC, because they both have a right angle (by definition of the altitude), and they share the angle at A, meaning that the third angle will be the same in both triangles as well. By a similar reasoning, the triangle CBH is also similar to *ABC*.

The similarities lead to the two ratios:

$$\frac{AC}{AB} = \frac{AH}{AC} \text{ and } \frac{CB}{AB} = \frac{HB}{CB}.$$

These can be written as:

$$AC^2 = AB \times AH \quad \text{and} \quad CB^2 = AB \times HB.$$

Summing these two equalities, we obtain:

$$AC^2 + CB^2 = AB \times AH + AB \times HB = AB \times (AH + HB) = AB^2.$$

In other words, the Pythagorean theorem:

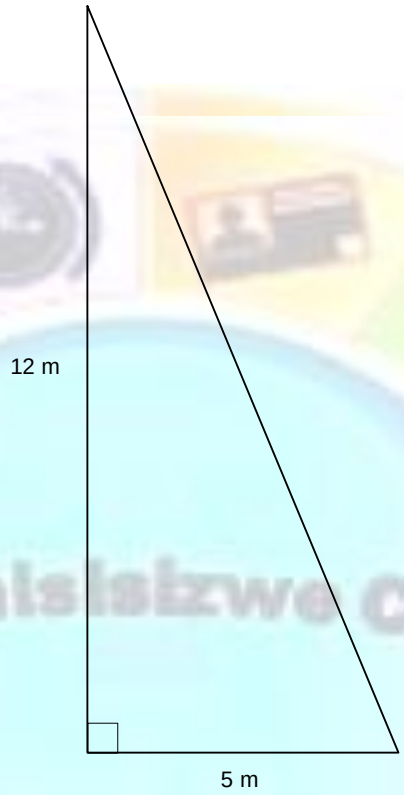
$$AC^2 + BC^2 = AB^2.$$

Formative Assessment

Calculate the hypotenuse of the right-angled triangle:

The hypotenuse is equal to the square root of the sum of the square of the other two sides:

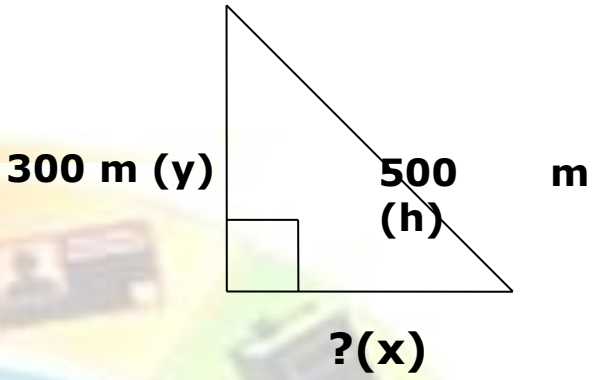
$A^2 = B^2 + C^2$. Therefore: $A^2 = 5^2 + 12^2 = 25 + 144 = 169$. Since $13 \times 13 = 169$, the hypotenuse is 13 metres.



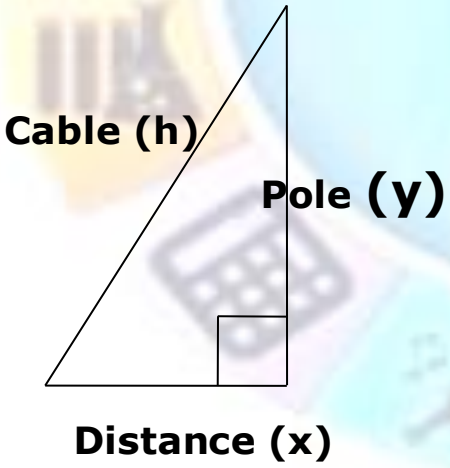
It is not always necessary to calculate the hypotenuse. Sometime the hypotenuse is a known value. In such case one of the right-angle sides might be an unknown length. Consider the following triangle.

Manipulate the equation of Pythagoras to get the unknown length on one side of the equation.

$$x^2 + y^2 = h^2$$
$$x^2 = h^2 - y^2$$
$$= 500^2 - 300^2$$
$$x = \sqrt{160000}$$
$$x = 400 \text{ m}$$



You are required to anchor a upright pole (***7 m high***) to the ground at four points around it. The anchor points must be at right angles to one another and ***5 m away from the pole***. The only measurement tools issued to you are a tape measure and a piece of string. Calculating the length of cable to cut for an anchor is important because the workshop is quite far away, so walking twice is out of the question. Cutting too much would increase waste cost. A side-on view would look like this.



First calculate the length of cable needed.

Given: Distance x = 5

Pole y = 7

Cable $h^2 = x^2 + y^2$

$= 5^2 + 7^2$

$= 25 + 49$

$= 74$

$h = \sqrt{74}$

$h = 8.6$ m

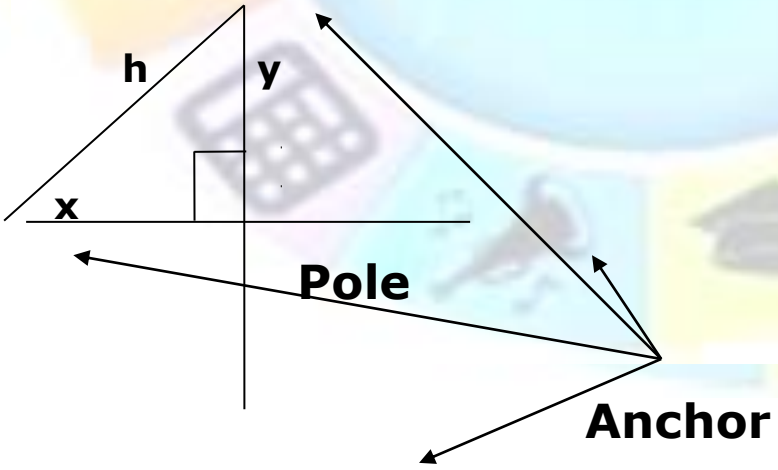
Since there are four cables

Total length to cut = 4 x 8.6

= 34.4 m

In practice you would add some cable to be tied.

Now look at the top view.



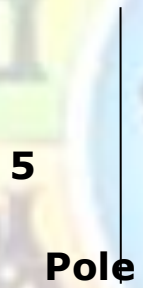
Knowing the distance from the pole to the anchor points we cut two pieces of string **5m** in length.

Using Pythagoras we can now calculate a hypotenuse. This would ensure a right angle between the other two strings.

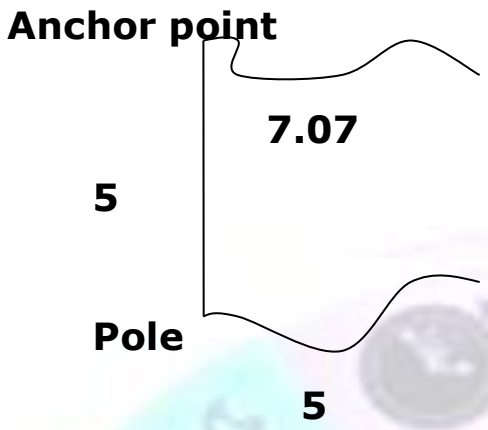
Given: Distance from pole to anchor x = y = 5 m

$$H^2 = x^2 + y^2$$
$$= 5^2 + 5^2$$
$$= 25 + 25$$
$$h = \sqrt{50}$$
$$= 7.07 \text{ m}$$

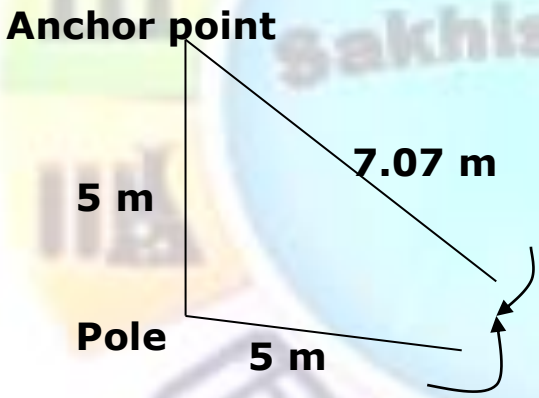
Now construct a triangle. First tie up one 5 m length string between the pole and first anchor point. The direction of this anchor point is not important because the other points will be placed with reference to this one.

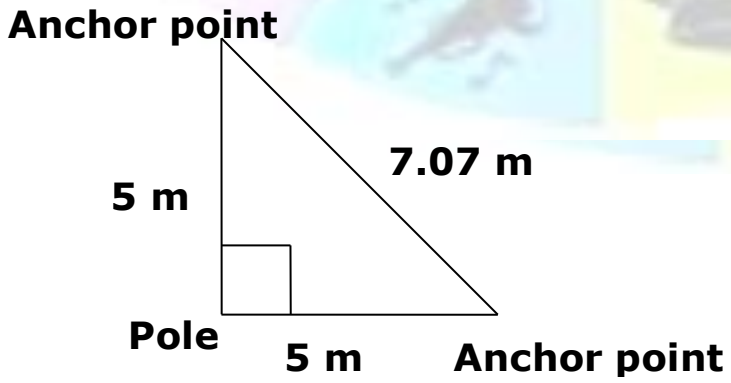


We assume a flat surface around the pole within the ***5m*** radius. Now tie one end of the other ***5m*** length string to the pole and one end of the ***7.07m*** string to the first anchor point.

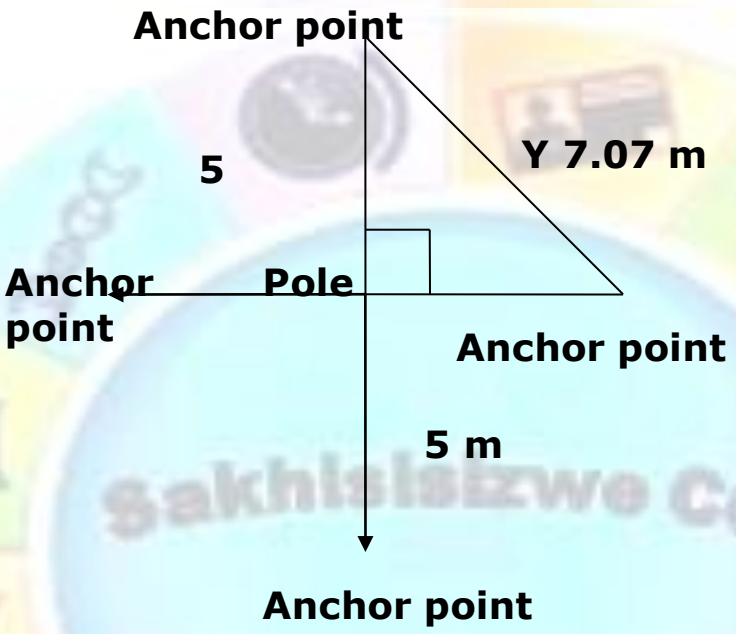


Pull both the ***7.07m*** string and the ***5m*** string straight. Keeping them tensioned, bring the two ends in your hands together.





This point should be the second anchor point. The rest of the anchor points can be obtained by repeating this process twice more. An alternative would be to extend each ***5m*** string through the pole point, in a straight line. ***5m*** down these extensions lie the other two anchor points. All the anchor points are now at right angles relative to their adjacent anchor points.



Formative assessment: using the SI system

The SI system uses the metric (decimal) system and uses a number of standard prefixes for units of length and mass that were covered in the previous section. Using the SI system means that we should know the most important ones. The three most important ones are:

$$\text{kilo} = 1000$$
$$\text{centi} = \frac{1}{100}$$
$$\text{milli} = \frac{1}{100}$$

Complete each of the following:

150cm = _____m
360mm = _____m
62ml = _____litres
3.6 tonnes = _____kg

Complete the table:

Quantity	Unit	Symbol
Mass		
	Meter	
		s
Temperature		
		A
Light		
Chemical standard unit		

WORK WITH GEOMETRIC SHAPES

Outcome

Explore, describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations.

Outcome Range

- ✓ Applications taken from different contexts such as packaging, arts, building construction, dressmaking.
- ✓ The use of tessellations and symmetry in artefacts and in architecture.
- ✓ Use rough sketches to interpret, represent and describe situations.
- ✓ Use and interpret scale drawings of plans (e.g., plans of houses or factories; technical diagrams of simple mechanical household or work related devices such as jacks,
- ✓ Nets of prisms and cylinders.
- ✓ Road maps relevant to the local community.
- ✓ The use of the Cartesian co-ordinate system in determining location and describing relationships in at least two dimensions.

Assessment criteria

- ✓ Descriptions are based on a systematic analysis of the shapes and reflect the properties of the shapes accurately, clearly and completely.
- ✓ Descriptions include quantitative information appropriate to the situation and need.
- ✓ Conjectures as appropriate to the situation, are based on well-planned investigations of geometrical properties.
- ✓ Representations of the problems are consistent with and appropriate to the problem context. The problems are represented comprehensively and in mathematical terms.
- ✓ Results are achieved through efficient and correct analysis and manipulation of representations.
- ✓ Problem-solving methods are presented clearly, logically and in mathematical terms.
- ✓ Solutions are correct and are interpreted and validated in terms of the context of the problem.

Two and Three Dimensional Geometric Situations

Symmetry

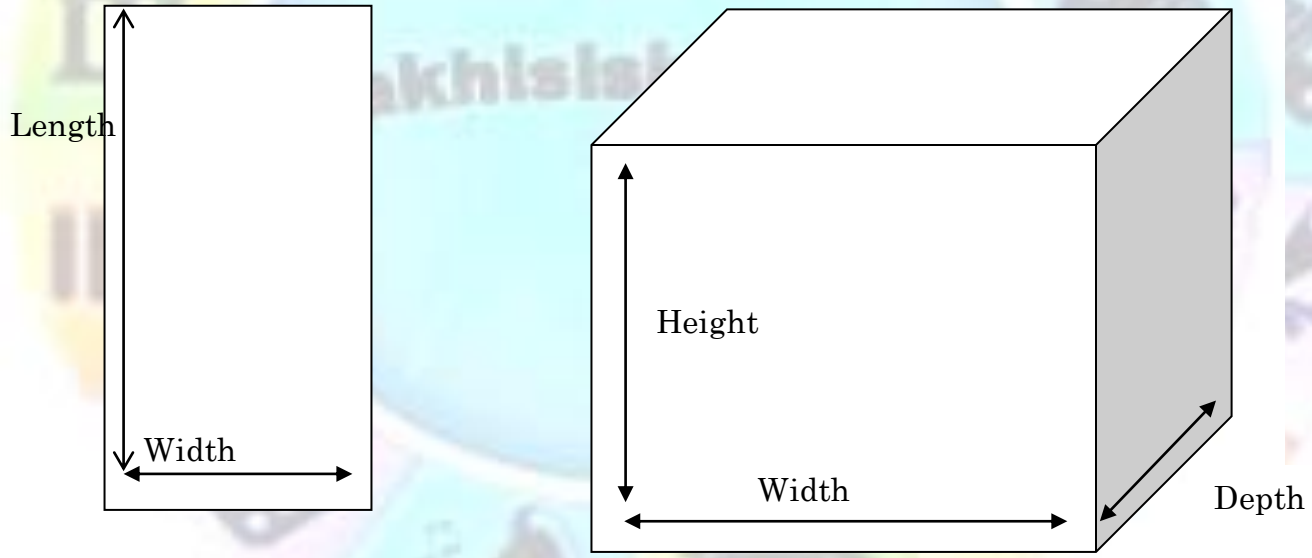
A symmetrical object is one that remains identical if rotated or reflected ('flipped') around a line through its centre. There may be many angles of rotation for an object.

Using symmetry reduces the amount of work you must do when calculating areas and volumes. Use symmetry to your advantage. If you draw an object that has symmetry, draw the portion you need then place copies in the correct places by rotating or reflecting them about their axis of symmetry.

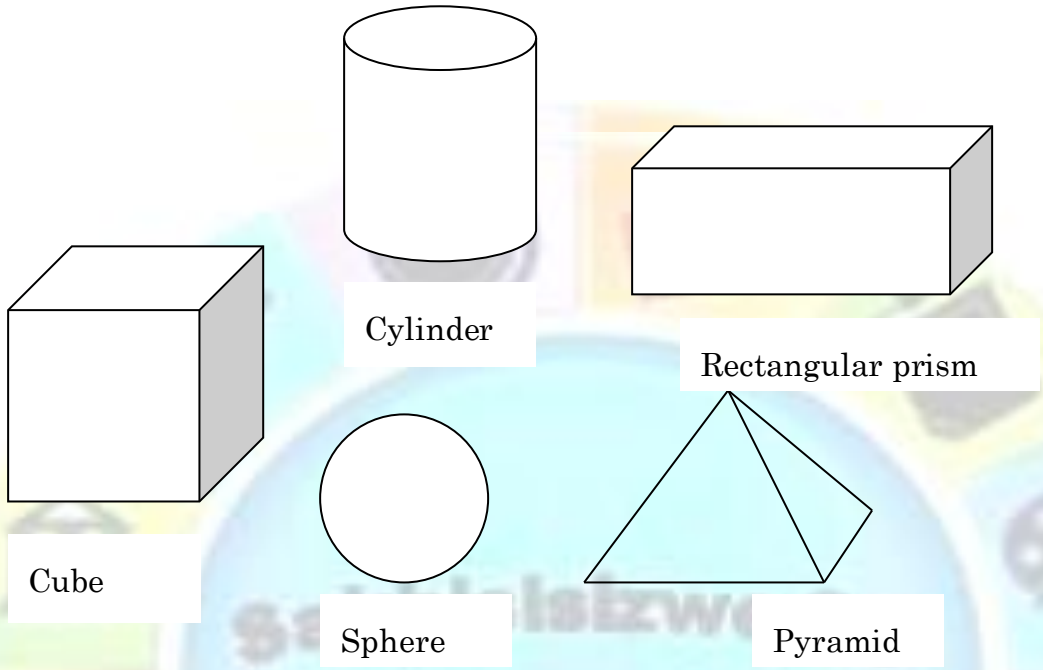
When we talk about seeing things in three dimensions, it means the following:

The first two dimensions are height (or length) and width on a flat surface. If you look at a rectangle, you have height (length) and width. A piece of paper has a length and a width that you can measure.

The third dimension is shown by introducing depth. A box has length, width and depth. The drawing shows a box shape in three dimensions: length, width and depth.



Geometric Shapes



Formative Assessment

Write down the names of objects you have come across in your daily life which have the shapes of the solids shown above.

Cylinder
Cube
Rectangular prism
Sphere
Pyramid

Surface Areas and Volumes of Right Prisms

In this section we will look at calculating surface areas and volumes of right prisms and other bodies.

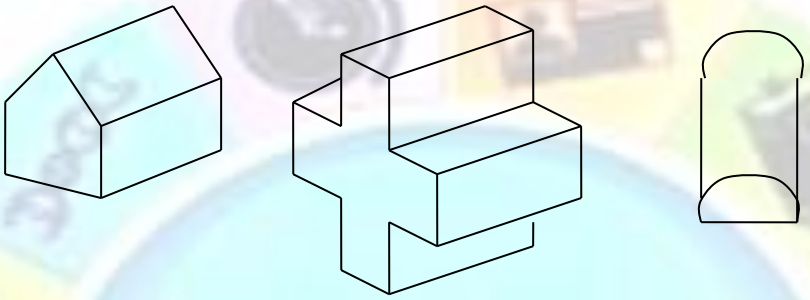
In all calculations the value of pi (π) should be 3.141593

A prism is a solid geometric figure whose two ends are parallel (side by side and having the same distance continuously between them) and of the same size and shape, and whose sides are parallelograms (a plane figure with four straight sides and opposite sides parallel).

The end faces consist of a compilation of known shapes such as triangles, rectangles and circles.

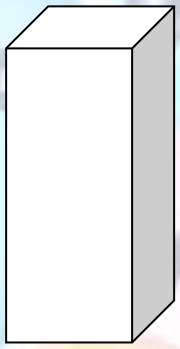
Each side surface is a rectangle. The following are possible types of shapes holding these characteristics.

It is easiest to calculate surfaces and volumes by breaking up each prism's face end into its most basic shapes. Let us review the surface area equations relevant to these basic shapes.



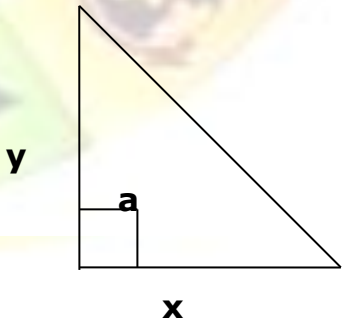
Right prism

This is what a right prism looks like



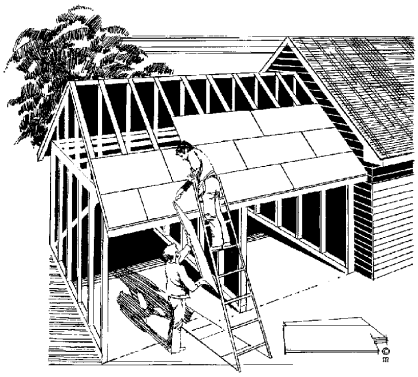
Right Triangle

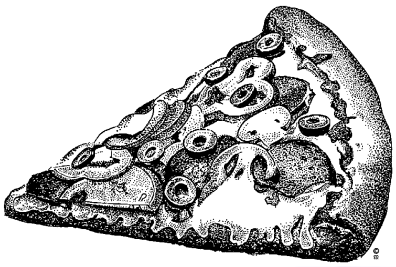
A plane figure with three straight sides and three angles: many houses have roofs that look like triangles.



Area(a) = ½x × y (unit: m ²)

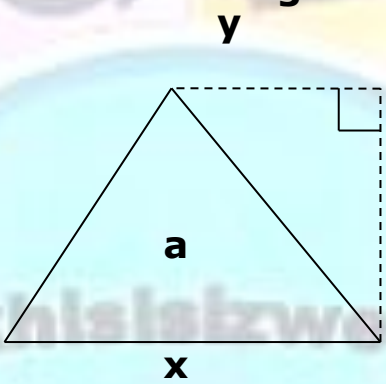
A slice of pizza or pie is usually in the shape of a triangle





Other Triangles

Area (a) = ½x × y



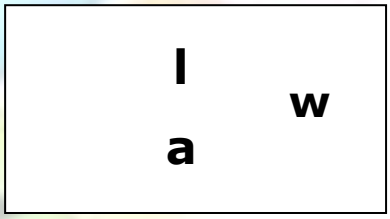
Rectangle and Square

A plane figure with four straight sides and four right angles and with unequal adjacent sides.

Area(a) = l × w (unit: m ²)

A square is a plane figure with four equal straight sides and four right angles

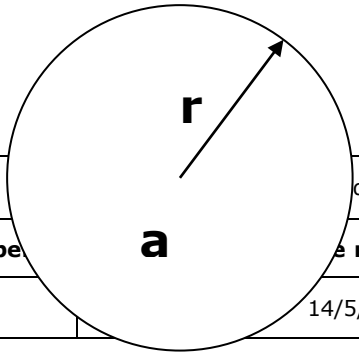
Note: We will treat the square as a rectangle with the same length and width.



Circle

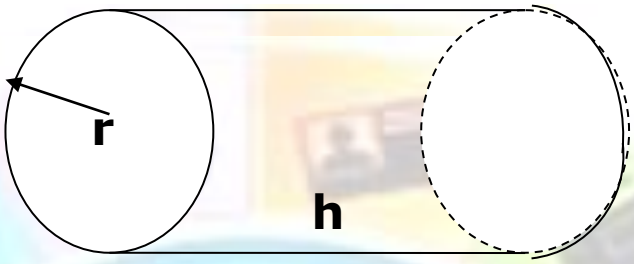
A round plane figure whose boundary is made up of points at an equal distance from the centre.

Area (a) = π × r ² (unit: m ²)



Cylinder

Three dimensional shape with straight parallel lines and circular or oval ends. A pipe is a good example of a cylinder.

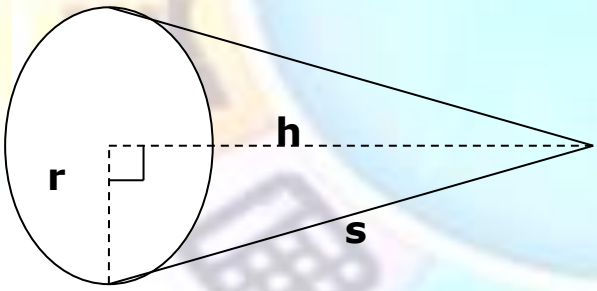


Volume = $\pi \times r^2 \times h$

Surface area = $(2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$

Cone

An object which tapers from a circular base to a point. An ice cream cone is a good example, although they would probably not be mathematically correct.

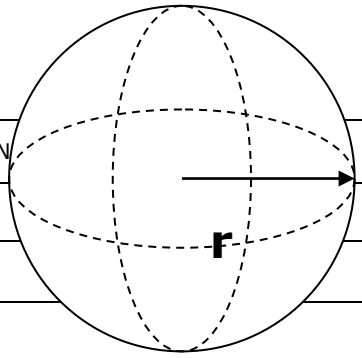


Volume = $\frac{1}{3} \times \pi \times r^2 \times h$

Surface area = $(\pi \times r \times s) + (\pi \times r^2)$

Sphere

A round solid figure in which every point on the surface is at an equal distance from the centre. A round ball is a good example.



Volume = 4/3 × π × r ³

Surface area = 4 × π × r ²

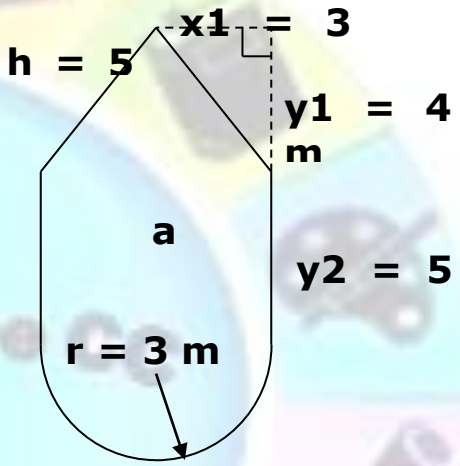
Pumpkins, oranges, apples, tomatoes and so on have spherical forms, although they are not exactly mathematically spherical.

Formative Assessment

We can now build a prism that consists of a combination of basic shapes.

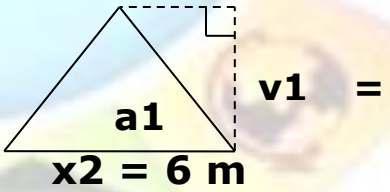
Viewed from the front it would look like this.

This shape can now be broken up into is most basic "sub" shapes.



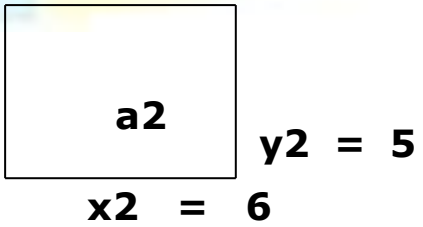
1. Triangle

Area (a1) = ½x2 × y1
= ½(6) × 4
= 3 × 4
= 12 m ²

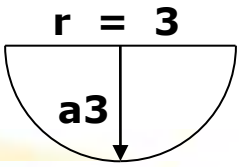


2. Rectangle

Area (a2) = x2 × y2
= 6 × 5
= 30 m ²



3. Half-circle

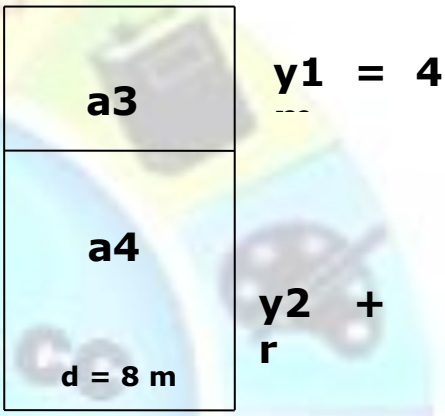


Area (a3) $= \frac{1}{2} \times \pi \times r^2$

$= \frac{1}{2} \times \pi \times (3)^2$

$= 14.14 \text{ m}^2$

To calculate the areas of the side surfaces we need a depth value. The side surfaces are all calculated as rectangular shapes. A side-on view would look like this.



Again we start with the triangle. The two angled surfaces ***(a3)*** would have the same areas because they have the same dimensions.

Area (a3) $= d \times h$

$= 8 \times 5$

$= 40 \text{ m}^2$

Since there are two surfaces with identical measurements this value is doubled.

Area(Triangle side surfaces) $= 2 \times$ Area(a3)

$= 80 \text{ m}^2$

Area **(a4)** consists of two parts namely the circular part and the straight rectangle part.

First we calculate the two surfaces forming the rectangle.

Area (Rectangle side Surfaces) $= 2 \times d \times y2$

$= 2 \times 8 \times 5$

$= 80 \text{ m}^2$

Lastly we calculate the area of the side surface forming the circular part.

Start by calculating the circumference of a full circle with a radius of ***3 m***.

Circumference $= 2 \times \pi \times r$

$= 2 \times \pi \times 3$

$= 18.85$ m

We use half this value because there is half a circle in the prism.

The area of the side surface would then be a rectangle with measurements 9.43 m and d = 8 m.

Area (Circular side surface) = 9.43×8

$= 75.44$ m ²

The total outside surface area can now be calculated. Remember that there are two face ends so all face end shapes will be doubled.

Total Area = 2 × (Area (a1) + Area (a2) + Area (a3)) + Area(Triangle side surfaces) + Area (Rectangle side Surfaces) + Area (Circular side surface)

$= 2 \times (12 \text{ m}^2 + 30 \text{ m}^2 + 14.14 \text{ m}^2) + 80 \text{ m}^2 + 80 \text{ m}^2 + 75.44 \text{ m}^2$

$= 347.72 \text{ m}^2$

To calculate the volume of this prism we will take the face-end surface area and multiply it by the depth.

Volume = (Area (a1) + Area (a2) + Area (a3)) × d

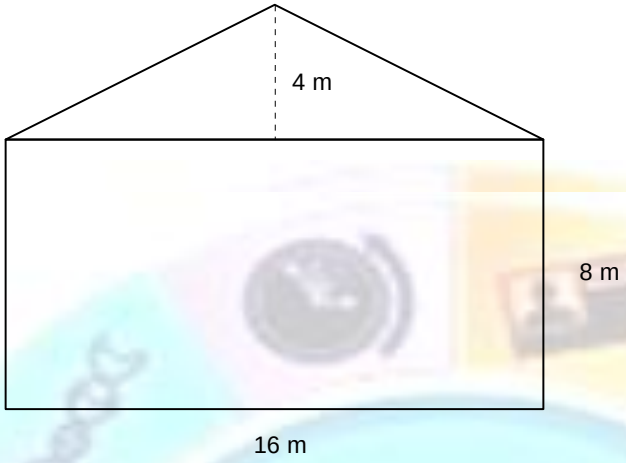
$= (12 \text{ m}^2 + 30 \text{ m}^2 + 14.14 \text{ m}^2) \times 8$

$= 449.12 \text{ m}^3$

Formative Assessment

Take 1 piece of A4 paper. Calculate the area. Calculate the circumference. What shape is the paper?
Fold the paper in half, so that it resembles A5 size paper. What shape is the paper now? Calculate the area. Calculate the circumference.

Which shapes have been combined to make this drawing?



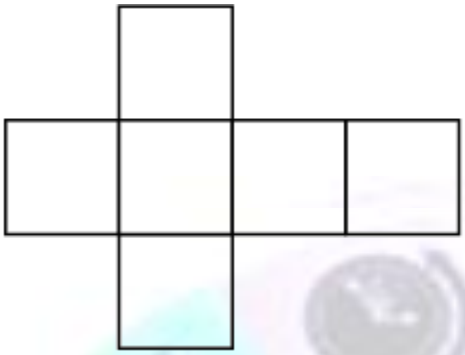
Draw a square where all the sides are 6cm long. Calculate the area. Calculate the circumference.
Draw a parallelogram where two of the sides are 6cm long and two sides are 30mm long.

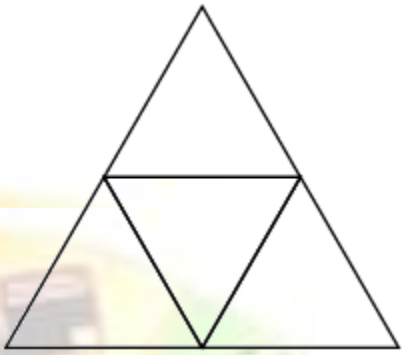
Nets

The word "net" has several meanings in mathematics. It refers to a plane diagram in which the polyhedron edges of a polyhedron are shown, a point set satisfying certain uniformity of distribution conditions, and a topological generalization of a sequence.

The net of a polyhedron is also known as a development, pattern, or planar net. The illustrations above show polyhedron nets for the cube and tetrahedron (triangular prism).

In English we can say that a net is a two dimensional plan of a geometric object e.g. a cube, prism, sphere, etc. If we fold this plan along certain lines, we will create an object. People working in the sheet metal industry make extensive use of nets to indicate where and in which order the different planes (sides) must be bent to form e.g. rainwater goods like gutters, offsets, down pipes, etc. or office furniture like steel filing cabinets, shelves, etc.

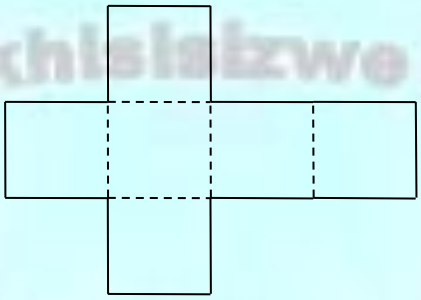




Formative Assessment

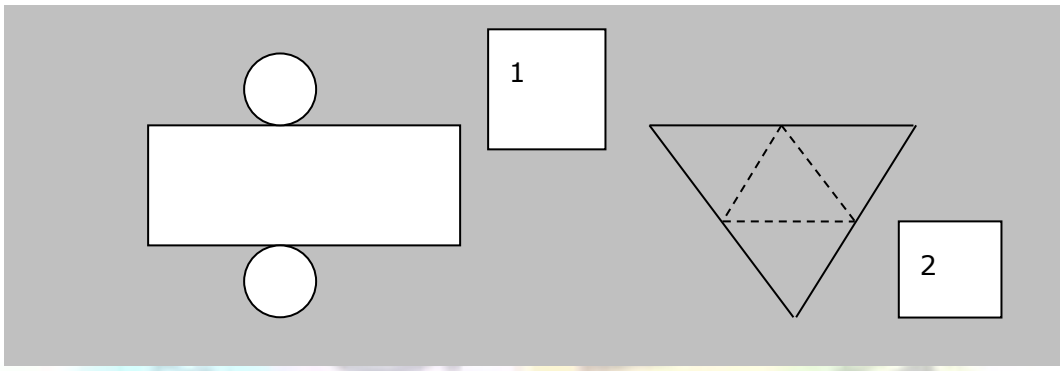
Draw the following figure on paper and fold along the broken lines.

All sides are 3 cm long. Which solid do you make, folding the figure? This figure is called the net of a cube.



Formative Assessment

1. Draw the net of a rectangular prism.
2. Which solids do the following nets represent?


1
2

Ratio

FIGURE A

FIGURE B

3 : 1

The above figures have the ***same shape*** but ***not the same size***.

There exists a mathematical relationship between the corresponding lengths on the two figures.

This relationship can be obtained as follows:

	Figure A (y mm)	Figure B (x mm)	$\frac{y}{x}$
Length of top of roof	6,0m	2,0m	3/1 = 3
Length of door	1,8m	0,6m	3/1 = 3
Length of total bus	6,6m	2,2m	3/1 = 3

Thus we find that y:x = 3:1 and that the ratio of y to x is as 3 is to 1.

Formative Assessment

- ✓ Construct triangle ABC with AB = 2 cm, BC = 3cm and AC = 4cm.

- ✓ Construct triangle DEF with sides 2 cm longer than the sides of triangle ABC.
- ✓ Construct triangle PQR with sides twice as long as the sides of ABC.
- ✓ Which of the two triangles DEF or PQR is an enlargement of ABC?

Scale Drawings And Scale Models

Maps, plans of buildings, design drawings of machinery, etc. are seldom drawn to full size, but are usually reduced in size. We call these scale drawings.

When a scale drawing is made or a scale model is built, the shape of the actual object must be retained, i.e. every dimension on the actual object must be multiplied by the same scale factor:

Length on actual object x k = corresponding length on scale drawing where k is the scale factor.

In scale drawings and scale models it is usual to refer to the scale factor as the scale.

Scale has the same meaning as scale factor. But where the scale factor is expressed as a fraction, for example, 2 or ½, the scale is usually given in colon notation, i.e. 2:1 or 1:2

Scale = length on drawing (model): length on actual object

Important:

In order to compare two quantities by division we must express them in the same unit. The result is a number without any unit of measurement.

It is preferable to write a ration in simplest form. A ratio is in simplest form if the numbers in the ration have no common factor.

Ratio, as the comparison of quantities, gives the number of times one quantity is contained in another.

For example:

6 cm:2 cm: = 3:1 means 6 cm is 3 times as long as 2 cm

600 mm:900 mm = 2:3 means 600 mm is 2/3 as long as 900 mm.

Formative Assessment

A model train is made to a scale of 1:50. If the length of the real train is 25 m, what is the length of the model?
If the model is 15 cm high, what is the height of the real train?
On the plan of a building every 2 cm represents 1 m on the actual building. What scale is used?

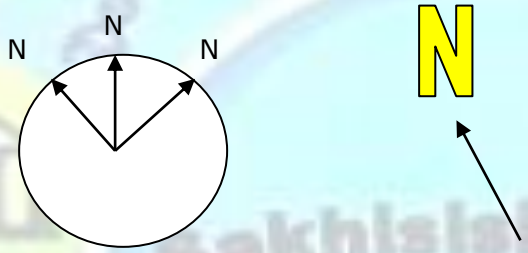
Rough Sketches

A rough sketch is a quick drawing of something that gives you a reasonable impression of a scene, object or surroundings but without much detail. The following is an example of a top view of a scene or incident that may be typical in a security situation.

A rough sketch is normally not according to scale but rather in proportion or in relation to size. This means that you may use a tape measure to indicate distances in relation to vital points or may even pace the distance between objects. The sketch may or may not be very accurate. However, the essentials have been captured in the sketch.

Some important elements must be displayed on such a rough sketch, such as:

- ✓ The direction north always pointing towards the top or at least like on a clock 10 to 2 or 10 past 10.



- ✓ The title "Rough sketch" on top of the drawing.
- ✓ The name of streets or buildings clearly displayed.
- ✓ Alphabetical numbering of critical elements on or at the scene if you are sketching a crime scene or incident scene.
- ✓ The name of the person drawing the sketch.
- ✓ The date and time of the sketch.
- ✓ Clear indication of grass, road surfaces and any other information that may assist the user of the sketch.
- ✓ Signature of the originator.

The sketch should have a separate sheet containing a key or explanation to the sketch. This we call the key or legend to the sketch. In the legend you set out measurements between points or distances.

Sketching in general

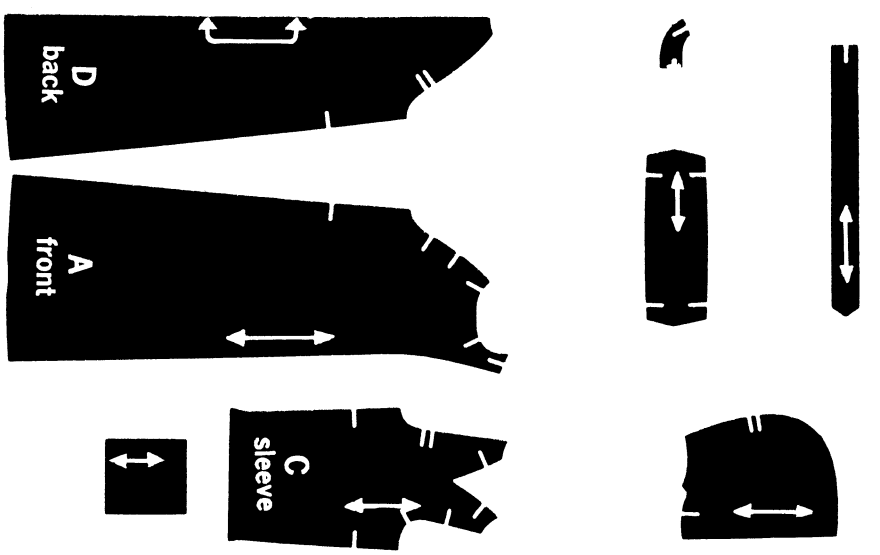
Sketches may be very rough, giving only a few small details, to very detailed enabling an item to be manufactured. The difference between a rough sketch and a sketch that is precise is not well defined. Additional information must be supplied with a sketch to provide enough detail to serve the purpose of the drawing.

So the 'roughness' of sketch may vary from a few lines drawn in the dirt with a stick to precision drawings used in fine engineering. A soccer team planning its strategy will sketch only the details required so that each player knows his function and the action he must take in order to

work as a team. The important thing to remember is that the detail that must be included in a sketch must suit the user of the sketch. The sketch must contain all the necessary information to convey the information required by the person using it.

Example

A woman making her own clothes or clothing for her children uses rough sketches to make the garment. Whether she draws the sketches herself or purchases them as a pattern in a shop, she still works with a rough drawing. A typical pattern for a girl's garment is shown below.



The 'documentation', 'report', or whatever you want to call it, consists of the metric measurements and information on how to layout, cut and sew the pieces together. As an aid to the seamstress who is making this garment, the original packet has illustrations of several variations of finished items.

Example

Imagine that you and your colleagues want to improve communications within your organization, church or local charity. In order to do this you decide that a monthly newsletter would help keep everyone in touch. In order to publish the newsletter you first want to get an idea of what the finished product would look like. You and your colleagues may discuss your needs but until you sketch a rough copy of its layout you really don't know what to expect.

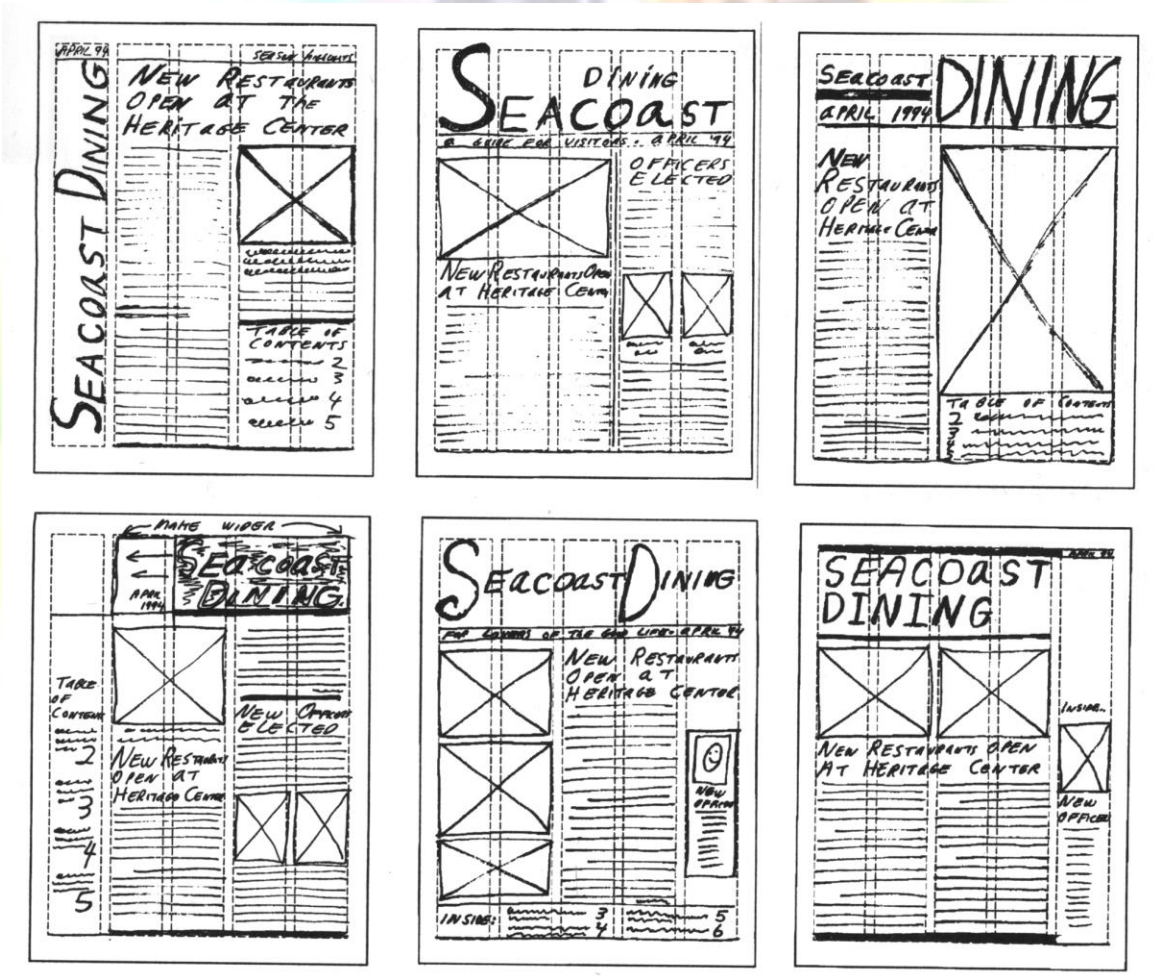
The documentation that accompanies these rough sketches would probably include the size of paper to use, whether the newsletter was folded or stapled, the use of one or both sides of the page, the number of columns and the font types and sizes to be used.

See the next page for the visual of the example.

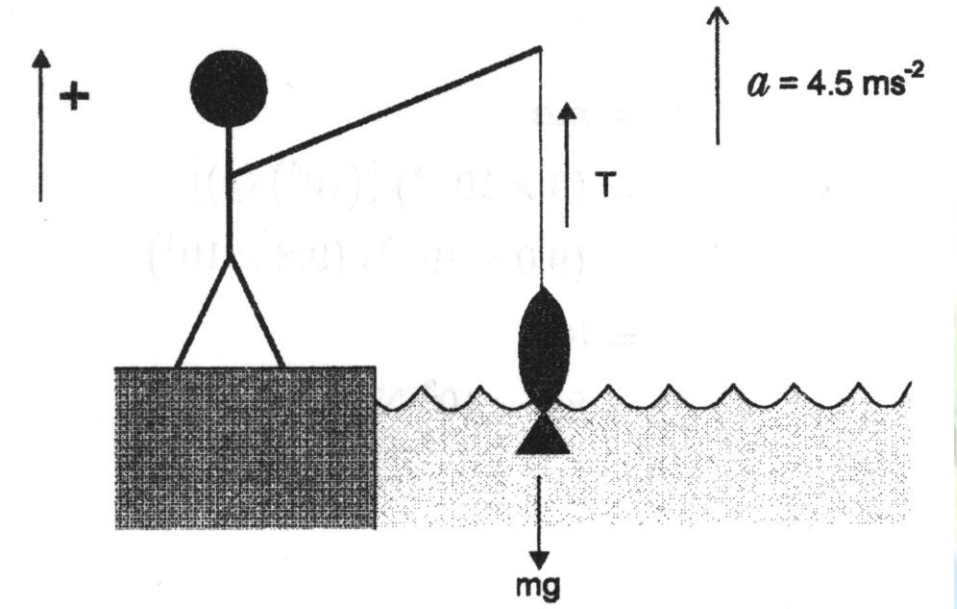
Example

A physics teacher might want to convey the action and reaction of forces and decides that a demonstration of a man fishing would be suitable. Below is the rough drawing that the teacher used to explain the concepts.

In this case the teacher does not need to show the person or fish in any detail nor does his scale need to be accurate. His 'report' would describe the forces involved. He would probably show his learners how to perform the calculations as well.



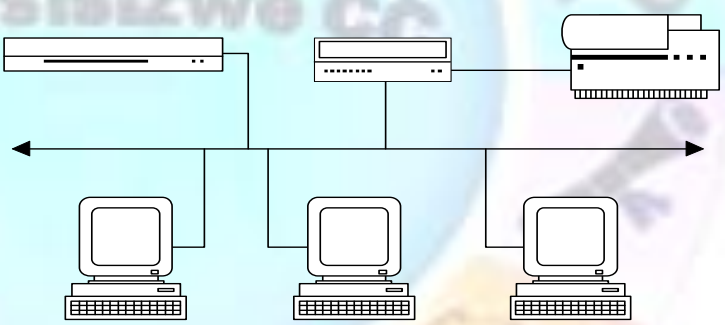
A ROUGH SKETCH OF A NEWSLETTER



A ROUGH SKETCH FOR A PHYSICS LESSON

Example

The concept of a computer network.

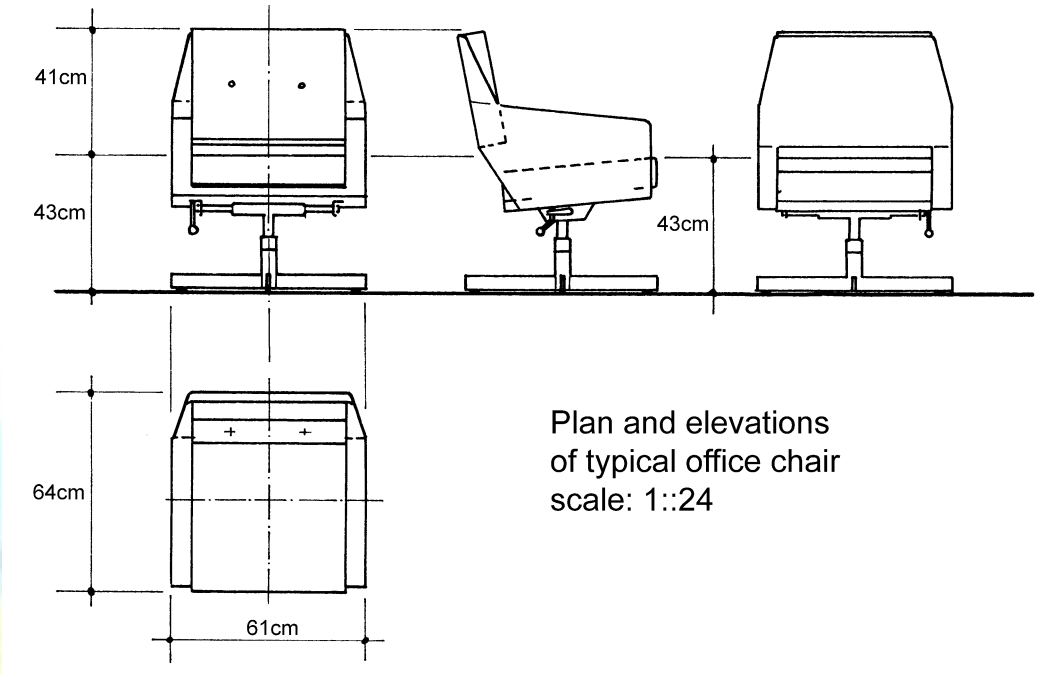


Scale Drawings

A scale drawing is a reduced or enlarged drawing of an original but it is drawn true to scale. Below is a scale drawing of a chair that was done on a computer. Notice how realistic it looks. Closer inspection will show that it is indeed a drawing and not a photograph.

Although it shows a realistic drawing of a chair, it may be considered a rough sketch by some. A manufacturer can't build the chair from this sketch. There are no scale or size measurements that go with the chair.

The difference between a realistic drawing and a rough sketch is determined by the user of the sketch. The person creating the sketch may put too little or too much detail in the drawing for it to satisfy the needs of the user of the sketch.



Another typical office chair that answers some of the criticism concerning the previous drawing.

Is this sketch better or worse than the previous one? Why? Can I build this chair in my factory? Why not?

Adding detail to scale drawings

In order to understand scale drawings it is a good idea to start from the known and proceed to the unknown. We are going to start with geometric shapes that are drawn to scale then proceed all the way to an introduction to engineering drawings.

The steps that take us from the rough to the precise involve four steps:

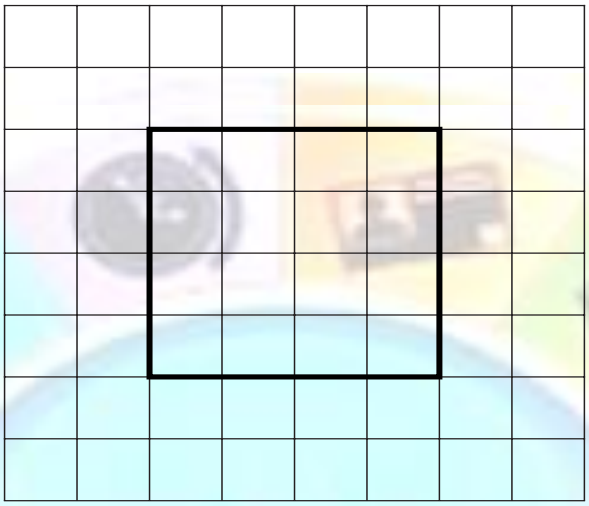
1. Learn how to make square and isometric drawings of geometrical shapes.
2. Learn what plans and elevations are when making drawings.
3. Learn what is meant by 'nets' of objects and use these nets to visualize and measure three-dimensional objects.
4. See examples of engineering drawings and the detail they contain.

Let's look at the simple geometric drawings that were used in previous sections. Two-dimensional items must be drawn to scale in order to appreciate what they are telling us.

There are several ways to represent two-dimensional objects. Annexure A contains a standard square grid while the second page contains an isometric grid. The first page is obviously a page of squares, but what is the second page a picture of? The second page is a pattern of triangles! Look at this page again and you will see that the dots make up triangles with the edges removed.

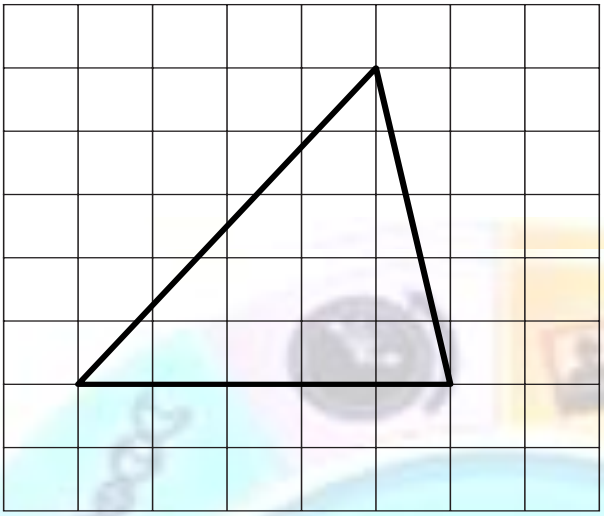
Using rectangular grids

Both square and isometric grids may be used to assist with 2D or 3D drawings. On the right is a square drawn on a square grid and the next figure shows a triangle drawn on a square grid.



Square grid paper is usually just referred to as grid paper. Some grid paper provides subdivisions that allow you to sketch very accurately. A popular grid is the millimetre grid that has very thin lines placed every millimetre and slightly thicker lines marking the centimetre. Some versions of the millimetre grid use slightly differently coloured lines to identify the different spacing.

Not only can you draw accurately using grid paper but you may use the grid to measure items as well. In addition, you may trace an item directly on the grid paper and have an accurate drawing that you can measure.



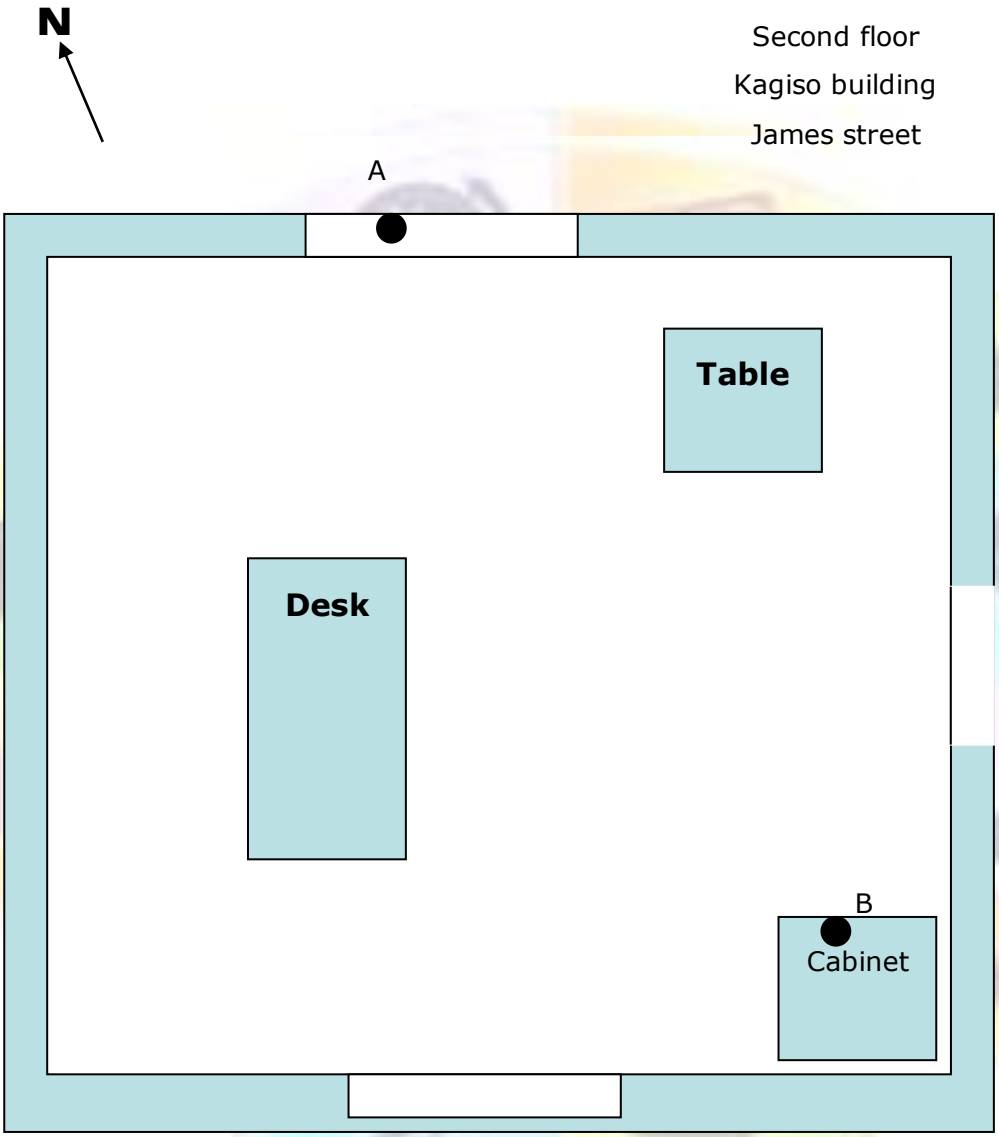
A triangle drawn on a square grid

What is the area of the square and the area of the triangle? The area of the square is 16 units^2 and the area of the triangle is 12.5 units^2. I use the term 'units' because I do not have any information concerning the size of each square.

These drawings may be scale drawings of real items or they might be the real sizes of these items. If the squares represent centimetres then the area is given in cm^2. If the squares represent metres then the areas of the items are measured in m^2.

Rough Sketch

Room 12
Second floor
Kagiso building
James street



Compiled by: SO John Dlamini
On 31 April 2005

Legend to sketch

Point A: Place of entry, broken window on northern side of office.

Point B: Location of cabinet (where money was stored).

Distances on rough sketch

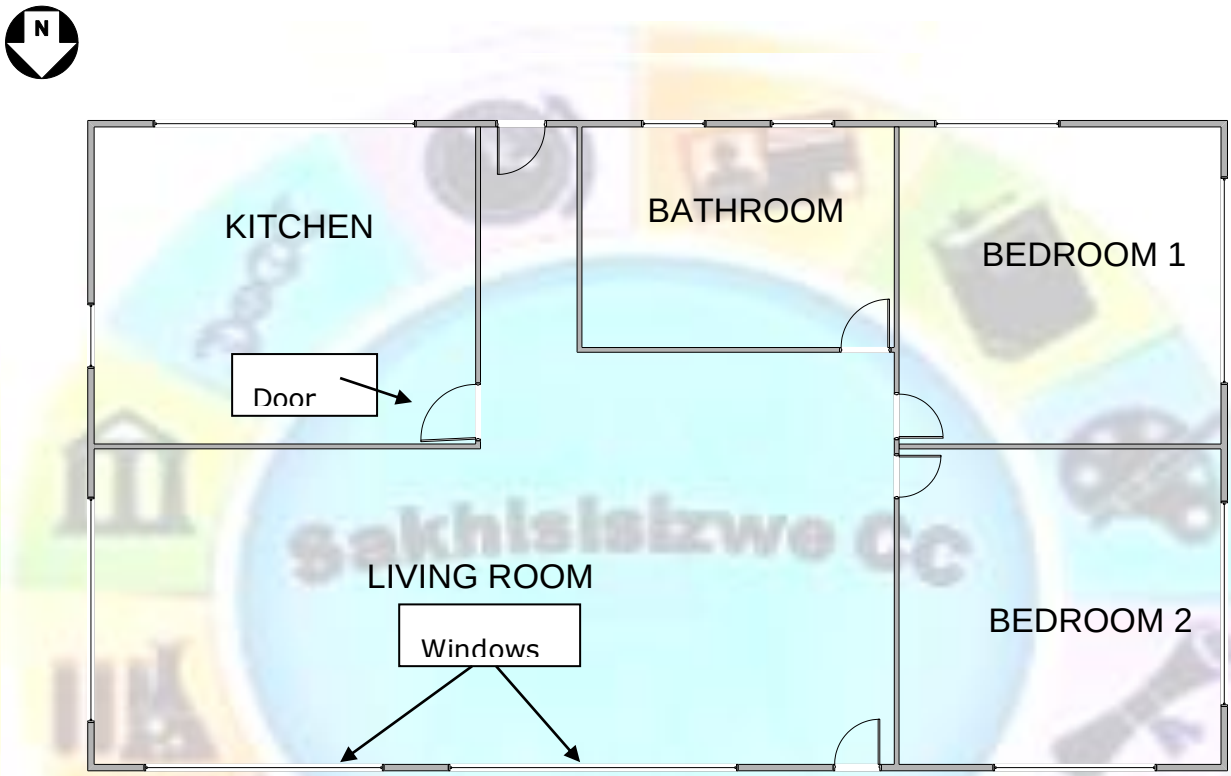
Point A to B: 2.4 m

With of room 12: 2 m

Length of room 12: 2.5 m

House Plans

Plans for houses are usually drawn flat, giving only the length and width of the area of the house and the individual rooms in the house.

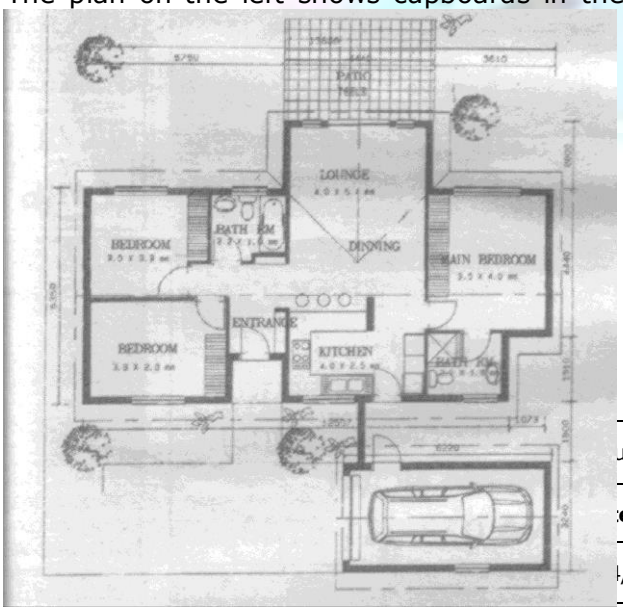


The figure above shows a basic plan, without any measurements.

This plan does not show built-in cupboards in the bedrooms, bath and basin in the bathroom or stove, sink, cupboards and fridge in the kitchen.

Handout 1

The plan on the left shows cupboards in the bedrooms, bath and basin in the bathrooms and stove, sink, fridge and cupboards in the kitchen.



This house also has a garage and the plan furthermore also shows details of shrubs and trees.

A house plan shows a view of the house from the top.

If you look at the kitchen, you will see a view of the sink and the stove as it will look when you look at it from the top. The same is true of the bath and basins, the cupboards as well as the car

in the garage and the shrubs and trees in the garden.

The bathroom adjoining the main bedroom does not have a bath, but a shower, a basin and a toilet. Can you see that the view of the shower is different to that of the bath in the other bathroom?

Of course, once a house is built we don't look at it from the top anymore, we look at it from the front or the side.

The houses on the left and right show a side view of a house with a patio, while the one in the middle shows a frontal view of a house with a patio and a water feature in front of it.

Once the house has been built, the view from the top looks different, since the roof has been added. This photo is, of course, of a very large house.

The purpose of the plan of the house is to give you an idea of the dimensions of the house and the individual rooms.

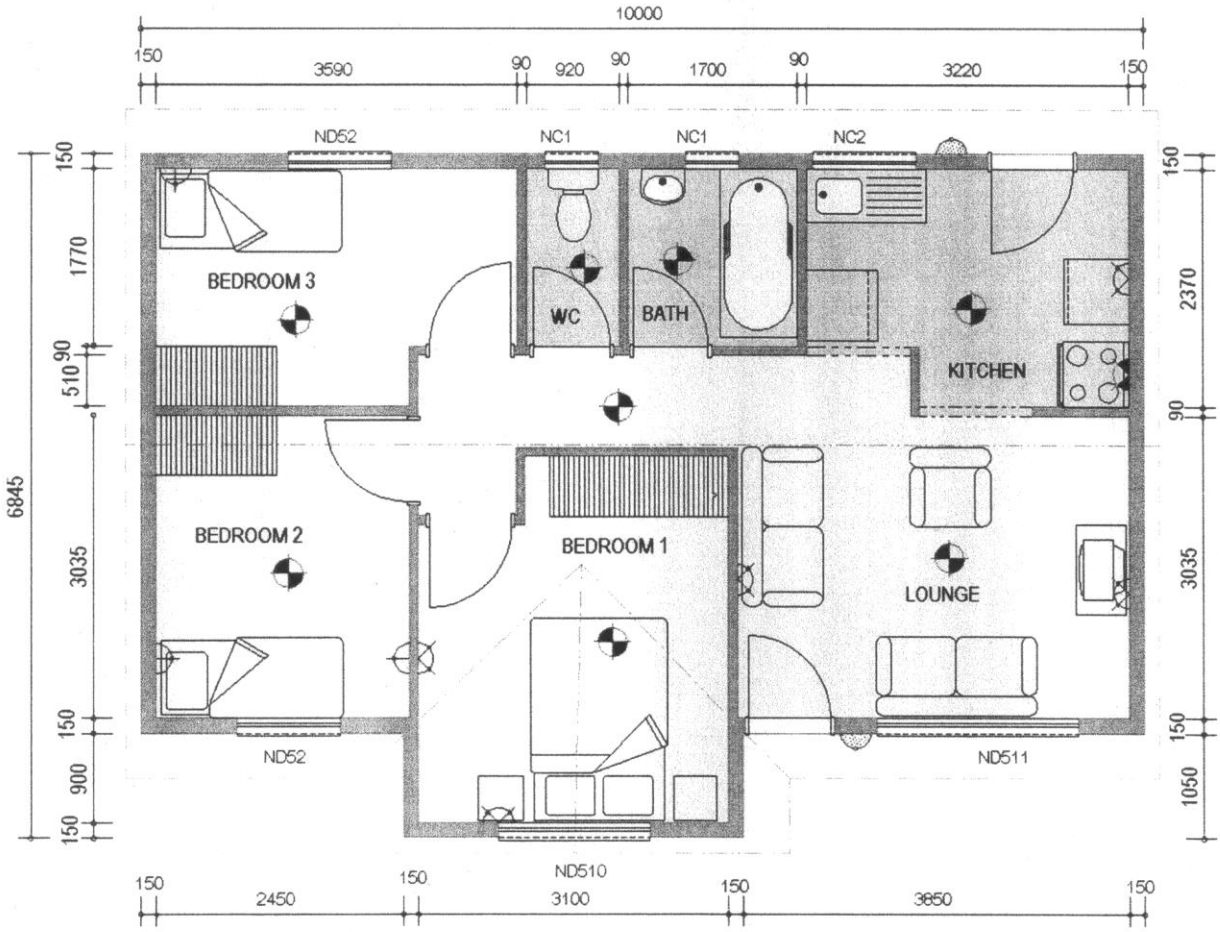
The house plan from the previous page has reasonably sized rooms, but it is not a large house like the one on the photo.

Of course, the builders also use the plans to build the house. They have to insert doors at the places indicated on the plan. The cupboards, bath, basins and kitchen cupboards and appliances are also built according to the plan.

You can also see from the plan that all the doors except the door from the garage to the courtyard open to the inside of the room. The door from the garage opens to the outside, as indicated on the plan.

Formative Assessment

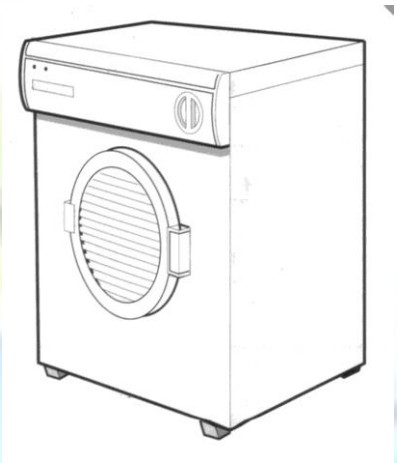
Look at the plan of the house on the next page. The area of the lounge is 3035cm X 3850cm. Calculate the area of the lounge in cm.
Convert the cm into m.


10000
150
3590
90
920
90
1700
90
3220
150
ND52
NC1
NC1
NC2
BEDROOM 3
WC
BATH
KITCHEN
BEDROOM 2
BEDROOM 1
LOUNGE
ND52
ND511
ND510
6845
150
1770
90
510
3035
150
900
150
2370
90
3035
150
1050
150
2450
150
3100
150
3850
150

Household Appliances

Household appliances always come with instruction booklets giving details of how and where the appliance should be installed. You must always first read the instructions before you install and use the appliance. The instruction booklet will include photos and drawings of the appliance to help you understand the instructions.

Also, before you buy an appliance such as a stove, fridge, freezer, washing machine or tumble dryer, you have to ensure that you have the space available for it. It will be silly to buy a big, double door fridge if your kitchen only has space for a single door fridge.



This is a picture of a tumble dryer. The view is a side view. The dimensions of the tumble dryer are as follows:

- ✓ Width 600 mm
- ✓ Depth 500 mm
- ✓ Height 850 mm

Formative Assessment

You want to place the tumble dryer under a shelf in the laundry.
The height of the shelf is 900mm. Will the tumble dryer fit under the shelf? Yes

Draw a front view of the tumble dryer with the door closed. Make sure that your drawing is according to scale, with a ratio of 10:1. If your drawing is not good enough or not right, do it over until you get it right. You will need it for the assessment. You do not have to draw the castors at the bottom of the tumble dryer. The dimensions of the drawing are:

Width: 60 mm

Depth: 50 mm

Height: 85 mm

Door: 40 mm wide and 40 mm high and 20 mm from the bottom

Front panel: 25 mm high and 60 mm wide

Packaging material

Boxes and cartons are the most commonly found packaging materials. They are also easy to make and decorate if you want to use them for gifts.

We will show you how to a gift box. The material you need is not expensive, for the first try you can make the box out of paper. It will not be as sturdy as using thin card paper, but is excellent to practice on.

Box With Overlap Lid

This box is based on a square and you can make the box in any size, as long as each side has the same width.

You will need:

- ✓ Thin card
- ✓ Tracing paper and carbon paper (optional)
- ✓ Ruler
- ✓ Pencil and eraser
- ✓ Glue
- ✓ Scissors

Handout 2

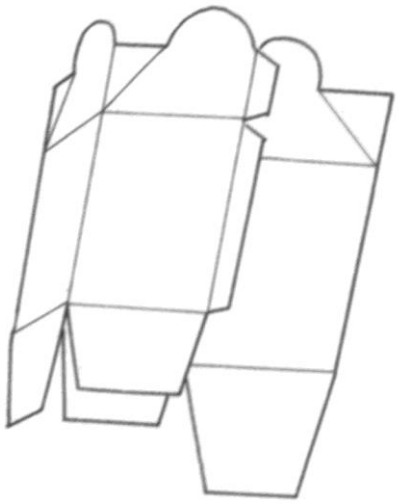
The box shape can be drawn directly on to card or transferred as follows:

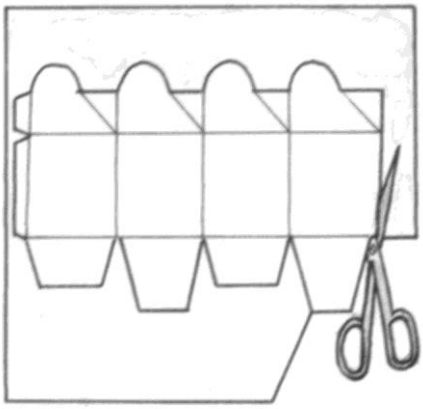
- ✓ take a tracing of the box and transfer on to graph paper.
- ✓ Scale up or down by copying the shapes on to a larger or smaller grid as required then take a tracing of the finished box shape.
- ✓ You can also enlarge or reduce your pattern on a photo copier.

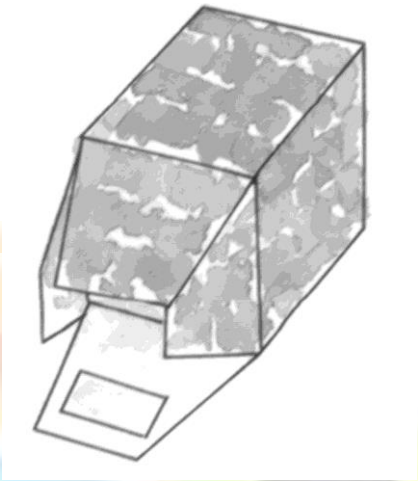
To make:

Handout 3

- ✓ Draw or trace the box shape on to paper (or card) and cut along straight lines and curves.
- ✓ Gently fold along all the fold lines to shape
- ✓ Erase any pencil lines as required
- ✓ Ease box into shape
- ✓ Run glue along the side flaps and press into position
- ✓ Fold in base flaps, shortest first
- ✓ Stick one large flap over the other with glue
- ✓ To close box, gently push top sections together and push flat







Formative Assessment

Handout 4

- ✓ Cut out the box in handout 4, follow the instructions above and make a box.
- ✓ Measure the height, length and width of the box and write it down.
- ✓ Calculate the volume of the box.
- ✓ Made a drawing of the finished box.

Cartography

Cartography is the practice of drawing maps. A map is a diagram of an area showing physical features, cities, roads, etc.

Maps vary in size from maps of the world, maps of continents maps of countries, maps of cities and even maps of shopping centres.

World Maps

Maps of the world come in more than one form:

- ✓ Political maps which give details of countries and capital cities of these countries
- ✓ Physical maps of the world that show mountains, major rivers, deserts or drier areas and tropical forests or areas that get more rain.

Maps are always drawn with north at the top, south at the bottom, east to the right and west to the left.

World maps are divided into latitudes and longitudes.

Latitudes

Latitudes start with the equator and divide the earth in horizontal bands north or south. The equator is at 0º, in other words in the middle of the earth.

The most common latitudes are the Tropic of Cancer and the arctic circle in the northern hemisphere and the Tropic of Capricorn in the southern hemisphere. Every year the sun moves from the equator north, causing summer in the northern hemisphere and winter in the southern hemisphere. When it reaches the Tropic of Cancer it reaches the northernmost part of its journey and it will be the longest day and shortest night in the northern hemisphere and the longest night and shortest day in the southern hemisphere. This happens on 21 June. Then the sun moves back to the southern hemisphere, reaching its southernmost point on 21 December every year. This is then the longest day and shortest night in the southern hemisphere and the shortest day and longest night in the northern hemisphere.

Of course, it's not really the sun that moves, but the earth that revolves around the sun and the angle at which the earth is aligned to the sun that changes. It is just common to talk about the sun moving north and south.

From the above it is clear that the latitudes have to do with seasons: summer, winter, etc.

Longitudes

Longitudes have to do with the earth revolving around its own axis and determine day and night and times. Longitudes divide the earth in vertical bands, from north to south, starting with 0º, which is situated at Greenwich and is known as Greenwich Mean Time. The longitudes then move in degrees east and west until they meet up at the other side of the earth at 180º. This longitude is known as the International Date Line.

This means that when the sun shines in South Africa, it is night in Australia and the USA.

The day starts in the western hemisphere, in Australia and Japan. When the sun rises there, it is the night of the previous day in South Africa and late afternoon of the previous day in the USA.

As the earth moves around its axis, it becomes late afternoon in Australia, early morning in South Africa and night of the previous day in the USA.

When it is late afternoon in South Africa, it is early morning in the USA and night in Australia.

These time differences are determined by latitudes.


NORTH AMERICA
EUROPE
ASIA
MIDDLE EAST
CENTRAL AMERICA-CARIBBEAN
AFRICA
SOUTH AMERICA
SOUTHEAST ASIA
OCEANIA
OCEANIA
ANTARCTICA

Map Of South Africa

Maps on a smaller scale, such as maps of countries and provinces, have legends where they explain the different symbols used, as well as the scale of the map.

The legend for this map is very basic, it only explains the scale of the map. However, we can deduce the following from the symbols on the map:

- ✓ Big cities
- ✓ International borders
- ✓ Provincial borders
- ✓ Names of provinces
- ✓ International airports

The map also shows the Orange and Vaal rivers.

Formative Assessment

List at least three cities in South Africa:
Which cities have international airports?:
Name two countries that are located to the north of South Africa:
South Africa encloses two other independent countries. Name them:
One province is not named on the map, which province is this?
The names of two provinces are different to that quoted on the map. Give their old and new names.

Cartesian Coordinates

Any position on earth can be specified by its latitude, longitude and height above sea level. For the purpose of this unit standard, we will focus only on two dimensions: latitude and longitude. Most coordinate readings are given as a series of numbers and letters as follows:

10° 05' 45"W

51° 28' 38"N

The first row indicates the longitude and the second the latitude. Both rows consists of 3 values, each followed by a sign.

The first value of the longitude indicates degrees away from Greenwich. As previously discussed, this value is divided into 180° in directions east or west. In our example, the degrees away from Greenwich is10º.

The second value of the longitude, 5′, indicates minutes. Note that in Cartesian coordinates this minute does not reflect minutes as we know them when telling time, but is merely one part in 60 of a degree. This means that each degree is divided into 60 minutes. In our example, the latitude is 10º and 5minutes.

The third value of the longitude, 45″, indicates the seconds. Once again, this value is not used as a time factor but as one part in 60 of a minute. In this case each minute is divided into 60 seconds. As per our example, 10º, 5minutes, 45 seconds.

The last letter, W, indicates whether the location is to the west or east of Greenwich, so our location is 10º, 5 minutes, 45 seconds west of Greenwich.

Now we have one half of our coordinates to find out where our location is: we have the exact latitude coordinates. Remember, a latitude runs from north to south across the surface of the earth, and we can be anywhere on that latitude. To plot ourselves exactly, we need the longitude

Latitude use the same division types as longitude except for the last letter which, in the case of latitude, indicates whether the location is to the north or south of the equator. In our example, 51° 28 minutes, 38 seconds north.

Now we will be able to plot ourselves accurately on any map.

Road Maps

Road maps can be countrywide, for a specific region or for a specific city or town. It stands to reason that countrywide road maps will only show major roads such as national roads (freeways), major provincial routes, minor provincial roads, etc.

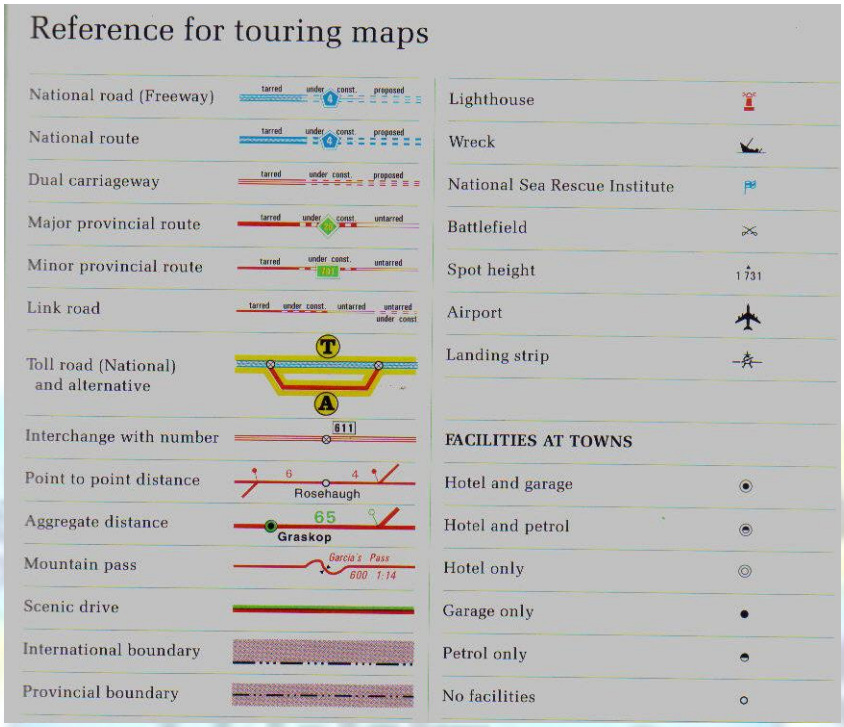
National roads or freeways are indicated with the colour blue, major provincial roads with a thick red line and minor provincial roads with a thin red line.

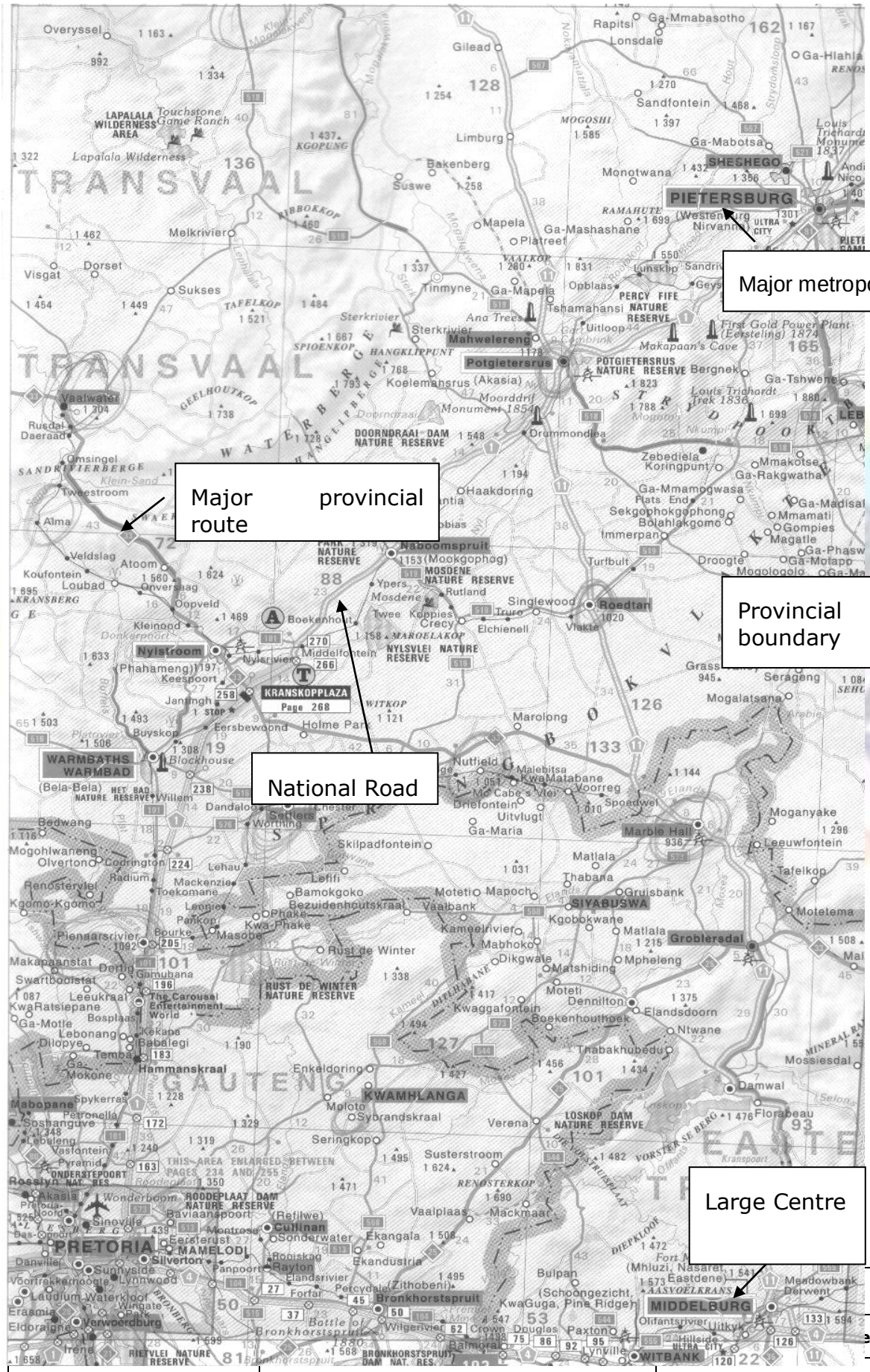
Each map will have a legend explaining the colour coding of the roads, the signs and other relevant information.

The legend below, taken from the Reader's Digest Book of the Road, has the following information:

National road	National route
Dual carriageway	Major provincial route
Minor provincial road	Link road
Toll road	Interchange with number
Point to point distance	Aggregate distance
Mountain pass	Scenic drive
International boundary	Provincial boundary
Lighthouse	Wreck
National Sea Rescue Institute	Battlefield
Spot height	Airport
Landing strip	A legend giving details about facilities available at towns

Of course, there is more information than quoted above.




Major metropolitan area
Major provincial route
Provincial boundary
National Road
Large Centre
PIETERSBURG
SHESHEGO
Mahwelereng
Potgietersrus
Naboomspruit
Roedtan
Nylstroom
KRANSKOPPLAZA
WARMBATHS WARMBAD
SIYABUSWA
Groblersdal
KWAMHLANGA
PRETORIA
MIDDELBURG
WITBANK
TRANSVAAL
GAUTENG

On the previous page is a portion of a road map, showing the route from Pretoria to Polokwane (Pietersburg.)

Regional road maps and road maps of cities and towns are more commonly used than countrywide road maps.

As with all other maps, the orientation of the map is always north at the top, south at the bottom, east to the right and west to the left. You will also always find a legend that explains how to use the map and what the symbols mean.

Handout 6

All road maps typically divide the area into sections, which are called pages. At the back of the map, you will find an index to street names, as well as an index to suburb names. Once you have the address of the place you want to go to, you look up the street name(s) in the index, where you will find the following information: page number and grid reference numbers. The grid reference numbers are quoted numerically for longitudinal references and alphabetical for latitudinal references:

If you are looking for Ben Steyn Street in Boksburg West, the references will be quoted as follows:

Ben Steyn Street	Boksburg West	113	DV 124
Street	Suburb	Page	Grid Ref.

If we look at page 1 of handout 6, which is also page 1 of the road map of the Witwatersrand, issued by Map Studio, 12th edition, you will find a full explanation of how to use the road map.

Reference Panel

At the top is an explanation of the reference panel found on the top of all the pages of the map. This map, incidentally, divides the Witwatersrand area into pages from 2 to 207, in total 206 pages, and covers the following area: from Midrand in the north, Nigel in the west, Randfontein in the east and Lenasia in the south. This is a very large area, that is why 206 pages are necessary to give a detailed and readable road map.

Key Plan

The key plan is a plan of all the pages that cover the entire area, an example is found on page 4 of the handout, included to give you an example of a key plan.

GPS Coordinates

The GPS (Global Positioning System) coordinates are quoted at the top and bottom of the pages. The GPS system is based on the Cartesian coordinates. The grid lines are at an interval of half a minute, which makes it easy to work out co-ordinates on the map.

Index

There is an explanation of how to use the index pages.

Grid Reference System

An explanation, also quoted above, of how the grid reference system works.

Legend

Once again, a legend that explains the colour coding of the roads and the symbols used in the maps.

There is also an indication of the scale of the map, in this case 1:20 000 (one to twenty thousand). Next to the scale indication is a scale legend, which gives you an indication of distance of the map compared to actual distance. In our map, every 5mm equals 100m or 1cm equals 200m.

Formative Assessment

Refer to page 5 of handout 6 and give the GPS coordinates of the following locations:

Corner of Bennit Avenue and Farrar Street:
Corner of Main Reef and Pretoria Roads
Corner of Morgan and Gayle Roads

UNIT STANDARD 9012

Unit Standard Title

Investigate life and work related problems using data and probabilities

NQF Level

3

Credits

5

Purpose

This Unit Standard is designed to provide credits towards the mathematical literacy requirement of the NQF at Level 3. The essential purposes of the mathematical literacy requirement are that, as the learner progresses with confidence through the levels, the learner will grow in: a confident, insightful use of mathematics in the management of the needs of everyday living to become a self-managing person

An understanding of mathematical applications that provides insight into the learner's present and future occupational experiences and so develop into a contributing worker The ability to voice a critical sensitivity to the role of mathematics in a democratic society and so become a participating citizen

People credited with this Unit Standard are able to:

- ✓ Pose questions, collect and organise data
- ✓ Represent and interpret data using various techniques to investigate real life and work problems.
- ✓ Use random events to explore and apply probability concepts in simple life and work related situations

Learning Assumptions

Learners accessing this Unit Standard should be competent in Mathematical Literacy and Communications at NQF level 2.

Specific Outcomes and Assessment Criteria

Specific Outcome 1: Pose questions, collect and organise data

Assessment Criteria

- ✓ Situations or issues that can be dealt with through statistical methods are identified correctly
- ✓ Variables contributing to a problem situation are identified and addressed in data gathering, e.g. crime is related to time of day and location

- ✓ Appropriate and efficient methods are used to collect, record and organise data.

Specific Outcome 2: Represent, analyse and interpret data using various techniques to investigate real-life and work problems

Assessment Criteria

- ✓ Graphical representations and numerical summaries are consistent with the data, are clear and appropriate to the situation and target audience
- ✓ Different representations of aspects of the data are compared to take a position on the issue
- ✓ Calculations and the use of statistics are correct and appropriate to the problem
- ✓ Interpretations of statistics are justified and applied to answer questions about the problem
- ✓ New questions that arise from the modelling of the data are discussed

Specific Outcome 3: Use random events to explore and apply, probability concepts in simple life and work related situations

Assessment Criteria

- ✓ Data are gathered, organised, sorted and classified in a suitable manner for further processing and analysis
- ✓ Probabilities are determined correctly
- ✓ Distinctions are correctly made between theoretical and experimental probabilities
- ✓ The outcomes of experiments and simulations are communicated clearly

Unit Standard Essential Embedded Knowledge

- ✓ Methods for collecting, organising data and calculating statistics
- ✓ The meaning of concepts such as centre and spread
- ✓ Techniques for statistically modelling a situation
- ✓ Random events, equal likelihood, probability

Critical cross-field outcomes

Upon successful completion of this course, the learner will be able to:

- ✓ Solve a variety of problems or take a position on issues related to the learner based on data, statistics and probability
- ✓ Collect, organise, and interpret data and statistics to make sense of adult situations
- ✓ Use everyday language and mathematical language to represent data, statistics and probability and to communicate conclusions
- ✓ Use mathematics to, describe and represent and interpret life or work related situations and to solve problems relevant to the learner

COLLECT AND ORGANISE DATA

Outcome

Pose questions, collect and organise data

Assessment criteria

- ✓ Situations or issues that can be dealt with through statistical methods are identified correctly
- ✓ Variables contributing to a problem situation are identified and addressed in data gathering
- ✓ Appropriate and efficient methods are used to collect, record and organise data
- ✓ Data samples are of adequate size and are representative of the population

The Role of Statistics when processing data

What is statistics? Most of us seem to become startled or frightened when the word 'statistics' is mentioned. This should not be so. We all use statistics in our daily lives. It is a natural part of ourselves that has allowed us to survive from the days of living in the dark caves of our ancestors to exploring our moon, other planets as well as the galaxy.

'But statistics means working with numbers and I am not good with numbers' you may say. It is true that statisticians work with numerical facts that they call data. However, these numerical facts are simply ways of

- ✓ describing,
- ✓ grouping and
- ✓ summarizing information so that it is more easily understood.

Keep in mind that all numbers used in this module are fictitious, that is, they are numbers I invented myself. If I use numbers that are really true, I will let you know. So if I say something has risen or fallen by 20%, I made that value up just as an example.

Here are a few examples that you undoubtedly have heard and may have some idea of what they mean. (Remember to ignore the actual numbers!)

- ✓ Think of our sportsmen and sportswomen. We discuss their average runs at the cricket crease, bowling rates and the athlete's best, worst and average times on the track. We know the average number of goals a striker makes in a season. How do we obtain this information? Why do we say one player is better than another? Can we compare a gymnast with a cyclist?
- ✓ We hear the weather forecaster say that there is a 20% probability of rain today. Perhaps based on this information we decide not to take an umbrella with us. If the weather forecaster said that the probability of rain today is 80%, would you leave the umbrella behind?
- ✓ The government announces that the rate of crime is down by 20%, inflation is up by 10%, unemployment is down to 28% and the CPI (Consumer Price Index) has risen 3 percentage points on a year-to-year basis. What does all this mean? How do they get this information? Are these figures accurate?
- ✓ You have heard that smoking is linked to lung cancer and that HIV is linked to AIDS and that the evidence for these statements is 'statistical'. What kind of evidence is 'statistical' evidence?
- ✓ A medical researcher claims that taking a certain tonic reduces the risk of heart attack. How can an experiment be designed to prove or disprove this statement? What is risk?
- ✓ You use statistics when you cross the street or drive a vehicle. When crossing the street you look both ways, see vehicles approaching, estimate their speeds, estimate your chance of making it across the street without being run over then you move or stay. If you move you have already decided how fast you will move based on the information you have just gathered and analysed. You behave similarly when in a vehicle. Our ancestors behaved similarly when looking for food.

- ✓ If you are a hunter or a sport-shooting enthusiast, you know how to aim and shoot your firearm. You also know that you rarely, if ever, hit the target in exactly the same place with every shot you take. Why is there a slight variation where the bullet hits the target? Why are the bullet holes on my target scattered, or spread, more than on another shooters target?

At this point you may have a few questions that you would like answered.

- ✓ What exactly do these figures mean?
- ✓ How did they (sport announcer, weather forecaster, government, researcher) obtain this information?
- ✓ How accurate is the information?
- ✓ What does it all mean?

We humans have survived for many thousands of years simply because we are born with the ability to take in information, organise and analyse it and then draw conclusions based on our analysis.

This is all that statistics is about:

- ✓ obtain information,
- ✓ organize and analyse the information
- ✓ and then draw conclusions from this information

The goal, however, of statistics is to provide insight by using numbers. In fact, the information usually contains some uncertainty but statistical thinking can deal with it.

Every discipline has developed its own language or terminology over time. Statistics is no different and many words and terms are used with specific meanings attached to them. Some of the same terms are also used daily by non-statisticians but in a loose, semi-defined manner. Statisticians use words to express specific ideas.

For instance, statisticians use the word 'data' where 'information' has been used in this introduction. The word 'data' is almost always used in the plural for statistical work and this module uses 'data' as a result.

The Use Of Statistics In Work Or Every Day Life

Statistics is the collection and analysis of numerical data in large quantities. This means that you gather information about a subject and then you analyse the information or data, so that you can distinguish trends. It is a very useful and easy way to "see" the story the numbers are telling.

Every time before an election, one of the organisations, Markinor, who collect and analyse data, will tell us before the election which political party will win the election and by how big a margin they will win the election. This is an example of gathering information and then analysing the information in order to find out what the trends are.

In the workplace, you can gather information about absenteeism at the workplace, how much fuel your vehicles use, how many employees are off sick during winter, how much stationery is used by the administration department, etc. Once you have the information, you can analyse it to find out what the trend is.

Research

Research is defined as all activities that provide information to guide business, societal and life decisions. Research is an information gathering activity that is intended to guide strategic or operational business, societal and life decisions about target groups, competitive strategies, etc.

Why would you want to do research?

Issues/questions that are commonly addressed by research in a business:

4 C's analysis:

- ✓ Customers (Customer analysis)
- ✓ Competitors (Competitive analysis)
- ✓ Company (Operational analysis)
- ✓ Climate (Environmental analysis)

Customer analysis

The following questions are common in customer analysis:

- ✓ How big is the existing market?
- ✓ How big is the potential market?
- ✓ How fast is the market growing?
- ✓ What are the buyer's background characteristics?
- ✓ How and why do buyers use the product?
- ✓ How and where is the product bought?
- ✓ How brand loyal are buyers?
- ✓ What market segments exists, and how large are the various segments?

Competitive analysis

In addition to analysing customers, market research may be used to describe your organisation's competitive position in the market. Relevant questions may include:

- ✓ What market share do you and various competitors hold?
- ✓ What future sales do you forecast?
- ✓ What are the awareness levels?
- ✓ How do buyers look at the different brands?
- ✓ What are the repurchase rates for the various brands?

- ✓ How satisfied are the customers with the various brands?
- ✓ What are your competitors' resources and strategies?

Operational analysis

Questions that arise under the heading of operational analysis include:

- ✓ How effective is your distribution?
- ✓ How effective is your advertising?
- ✓ How effective are your sales promotions?
- ✓ How effective are your sales people?
- ✓ How effective are your pricing strategies?
- ✓ How might consumers respond to product changes?
- ✓ How might buyers respond to a new product?

The value of research

Research derives its value from helping managers to make better decisions. It does not change the outcomes of those decisions: it simply helps managers know which course of action is best. Therefore, the value of research in any given situation depends on the importance of the decision at issue, the level of uncertainty about the proper course of action and the ability of the research to reduce that uncertainty.

Where would we get the information required for research?

Sales and expense records

Sales and expense records take two forms. The first is the traditional accounting compilations used to prepare income statements for an organisation's operating units. The second is sales and expense information organized by customer groups and not by operating units. The uses of traditional sales and expense data for operating units include:

- ✓ Sales data for a product or business unit can be analysed to measure seasonal fluctuations and make short term sales forecasts
- ✓ Sales for a product can be correlated with prices to estimate to estimate the price elasticity of demand.
- ✓ It can also be correlated with advertising expenditure to estimate the advertising response function
- ✓ Sales can be compared before, during and after a promotion to measure the effects of promotion.

Sales and expense data based on customers rather than operating units are less traditional but lie at the heart of database business, societal and life, which involves using purchase records and background data on individual customers to tailor what is offered to them or to develop target profiles for potential new customers. Example of using internal information in this way include:

- ✓ The profitability of an individual customer or a group of customers can be calculated to determine whether price concessions or business, societal and life expenditures are justified.
- ✓ Heavy purchasers can be compared with light purchasers to develop a profile of key target customers
- ✓ A customer purchase records can be analysed to estimate that customers buying cycle, so that purchase reminders can be sent when a customer is due for a purchase. This also enables companies to send promotional incentives to these customers.

The problem with using sales and expense records is the difference between what has happened and what is yet to come. They provide information on what happed in the past, but future conditions may be different. The past is not always the best prediction for the future.

Sales peoples reports

Four types of reports can be useful:

- ✓ Request and information reports show customer reports that can not be fulfilled and customer complaints. Documenting these, helps the company recognize problems and opportunities.
- ✓ Lost sales reports provide information on lost sales opportunities. This can alert management to trends and patterns
- ✓ Call reports show the date and time and dates of sales call, the company and person visited, the issues discussed, and the out come of the visit.
- ✓ Activity reports summarise a sales person's activities over some time period, how many calls were made, to whom and on what dates. The biggest limitation on using sales peoples reports is the workload they put on the sales for a. On the other hand imposing sales reports on sales people can cause resentment.

Street news

Street news about customers and competitors activities is another source of internal information. It should be made a regular part of a company's business, societal and life information system. This can be stabled by:

- ✓ Considering the types of information required
- ✓ Communicating these guidelines to people in the organization and establishing a reporting system.
- ✓ Regularly analysing and reporting this information

Surveys

When you do a survey you gather data by means of structured interviews. People who conduct surveys usually use a standardised questionnaire, which has certain advantages and disadvantages over structured interviews. The advantages of surveys are:

- ✓ The use of structured questionnaires makes it easy to analyse the data since all respondents (people who complete the questionnaires) are asked the same question in the same order.
- ✓ It allows the researcher to control the interview without being present
- ✓ It allows structured questionnaires as well as survey interviews to be done by telephone or through mail, which means they can be cheaper than interviews which require personal interaction.
- ✓ The use of telephone or mail and the lower costs per interview makes it possible to conduct a large number of interviews with a broader cross-section of the market.

The disadvantages of surveys are:

- ✓ Structured interviews reduce flexibility
- ✓ Deep feeling and hidden ones can not be probed very well
- ✓ Question are limited to those that provide short answers.

In conclusion, surveys are good for measuring facts but less so for in-depth studies or profiles of individual respondents.

Personal surveys, telephone surveys, and mail surveys

- ✓ Personal surveys offer maximum questionnaire flexibility.
- ✓ Personal surveys are used where telephone surveys are not appropriate.
- ✓ Intercept surveys, usually conducted in shopping malls, allow objects to be shown to respondents at a lower cost than in personal interviews.
- ✓ Sample quality is low.
- ✓ Save time with short questionnaire
- ✓ Telephone surveys offers a good sample quality
- ✓ The cost of telephone surveys is low.
- ✓ Mail surveys offer the advantage of lower costs

- ✓ Low response rate with mail surveys

Conducting A Survey (Research)

The basic process of conducting a market survey is as follows:

- ✓ Determine the aim of the survey (also called research)
- ✓ Identify the population and sample: from which population group will you gather the information?
- ✓ Decide how to collect replies: how will you get their replies to your questionnaire: by phone, mail, verbally, etc.?
- ✓ Design your questionnaire: the questions you will ask in order to collect the information
- ✓ Run a pilot survey: a test survey to check the process, the questions, the aim and the information
- ✓ Carry out the main survey: the actual research
- ✓ Analyse the data: now you will analyse the data in order to find out what it tells you.

It is important that the questions in your questionnaire asks questions that relate to the aim of the research in the first place.

This is why you must first determine the aim and identify the population sample and also decide

how you will collect the replies before you draw up the questionnaire.

You must also carefully think about how you will analyse the data before you start collecting the data. You should actually know before you even do the questionnaire how you will analyse the data.

Determine Your Research Aims

Start your survey by setting sown the aims for the survey. Why are you doing research and what do you want to achieve? What do you want to know? If we use Markinor as an example, they want to determine before the election who is going to win in which area, and how the other political parties will do during the elections.

In the workplace it can be that you want to find out:

- ✓ How many passengers you transport per route
- ✓ Why customers use your organisation rather than one of the opponents
- ✓ How much fuel your vehicles use
- ✓ How many man hours are lost every year during the winter due to illness of staff members

- ✓ How you can improve your customer service
- ✓ What other services customers require from your organisation

Doing a survey does not have to be difficult and complicated or expensive.

There is a story about a construction company that was trying to find a competitive edge. The management decided to do market research and so it asked customers about the worst habits of the competitors. Of course, the customers talked about the bad habits of constructions companies:

- ✓ Being impolite
- ✓ Don't care about the dirt that workers bring into the home
- ✓ Staff and equipment that looked shoddy

So, what did this construction company do to be better than their competitors? They

- ✓ Bought new equipment and kept it in good condition
- ✓ Trained their workers to be polite
- ✓ Dressed the workers well in order to project a good image

Did the company benefit from the market research and the changes that were brought in as a result of the market research? Yes. In less than two years the company increased its yearly sales FIVE TIMES!

Formative assessment

In a group, decide on a topic that you want to do research on. The topic has to relate to your place of work. Write down the aim of the research.

Identify The Population And Sample

Sampling is a basic concept used in statistics and is used to try to estimate what the parameters of a population are. For example, I may want to estimate how tall South African men are on the average. Perhaps, I measure a few hundred men and calculate their average. The men chosen for measurement are the sample. The parameter I am looking for is the average height of South African men.

One sip of milk is sufficient to let you know that the milk is sour. This is what sampling is all about. In order gain information about the whole you only need to examine a part. On the other hand, sipping a glass of water tells me nothing about the milk.

Here are a few definitions that statisticians use when dealing with samples:

- ✓ A '***population***' is the entire group of objects about which information is wanted.
- ✓ A '***unit***' is any individual member of the population.
- ✓ A '***sample***' is a part or subset of the population used to gain information about the whole.
- ✓ A '***sampling frame***' is the list or units from which the sample is chosen.
- ✓ A '***variable***' is a characteristic of a unit that is to be measured for those units in the sample.

The distinction between population and sample is extremely important to statistics. First, we look at a population then some examples to make the distinction clear.

A population is defined in terms of our desire for information about that population. For example, if I want information about all high school students in South Africa, then the population is all high school students in South Africa. Even if I can only choose one high school and its students, the population remains all high school students in South Africa.

It is extremely important to define clearly the population of interest.

If you want to determine how many South Africans are in favour of gun control laws, you must define the population precisely. Are all South African residents included, or only citizens? What is the minimum age you require? Are individuals imprisoned for violent crimes allowed to be included, or just those with minor offences, or none at all?

The following examples are presented to give an understanding of population, sample and variables. Data (numbers) used in most examples are completely fictitious as you have been warned.

Example 1

The national census attempts to collect basic detailed information from each household in the country.

- ✓ Population: all South African households
- ✓ Sample: the entire population, as far as possible.
- ✓ Variables: the number of occupants, age, race, gender, family relationships, access to electricity, water and telephone services.

Example 2

You want to do market research to find out your customers'preference and usage of various products or services. Undoubtedly you have heard radio or TV announcers state that their station is listened to or watched by 80% of South African while their competitors only make up the remainder.

- ✓ Population: all South Africans who have either a radio or a TV.
- ✓ Sample: about 1500 residents of South Africa

- ✓ Variables: gender, racial group, age, residential area, income category and the answers to the specific question or questions.

When Markinor does a survey before an election, they do not ask every person in the country for their views or opinions, they choose a number of people from the various population groups. The number of people they choose is called a ***sample***: a sub-set of the population, while the ***population*** is the members of the group you are interested in.

When you choose the sample for a countrywide survey, you have to make sure that your sample represents the entire population. Usually the ***sample*** will then be chosen from a list that contains all the members of the population, such a list is called a ***sampling frame***.

This is probably what Markinor does when they do surveys before elections. Luckily, for most of the research we want to do we do not have to go countrywide: we can usually choose from our community or customers or the customers of our competitors.

Market segmentation

When we determine the population sample, we usually want to know market segmentation as well. Market segmentation is a breakdown of your sample taking into account the following factors:

- ✓ Geographic factors: where do they live? If our business is in Gauteng, getting information from people in Cape Town will not help us at all
- ✓ Age: which age group should we target?
- ✓ Income bracket: what should their earning be?
- ✓ Gender: male or female or both?
- ✓ Community and cultural beliefs: how will this influence our survey?

If you need more information about population groups and samples, you can contact

- ✓ The government statistical services and
- ✓ the Department of Trade and Industry who will be able to give you information regarding the population segments according to the above criteria.

In order to find out how big your sample size should be (how many people you should question) you must first determine how many responses (completed questionnaires) you will need for the analysis.

A general rule is to look for about 20-30 responses in each of the major sub-categories for the sample. For example, if a key aspect of your research is to compare male and female then you should look for about 30 females and 30 males in your responses.

Once you know how many responses you want, 60 per the

above example, you have to find out how many questionnaires you have to send out. Usually, about 20% of the people will reply to the questionnaire. This means that you then have to send out about 300 questionnaires to get 60 responses (people replying to your questionnaire)

On the other hand, if you are going to interview your customers by telephone or face-to-face, you will need less questionnaires.

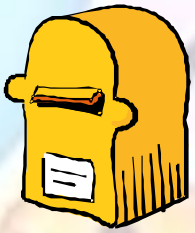
Formative assessment

In a group, determine the population group that you will target for your survey. Then determine the size of the sample as well as the market segment.

Decide How To Collect Replies

Now you have to decide the following:

- ✓ Are you going to mail the questionnaire
- ✓ Are you going to send the questionnaire by e-mail
- ✓ Are you going to phone customers
- ✓ Are you going to question customers face-to-face



If you are going to conduct the survey by means of telephone or face-to-face interviews, you have to decide who will do the interviews and how you will control the quality of the interviews. Remember, if you do not do this yourself, people can lie about how many interviews they conducted and what the response of the people they interviewed were.

It will also be important for the interviewer to explain to the potential respondent why they should answer the questions. They should persuade people to take part in the survey, not force them.

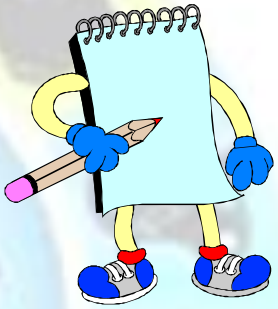
Formative assessment

In a group, decide how you will collect the information for your research project and who will do it.

Questionnaire Design

Most researchers make the mistake of asking too many questions. Your greatest enemy in survey research may well be poor response rate. Clear and concise questionnaires can help get the best response.

Design of the questionnaire can be split into three elements:



- ✓ Determine the questions to be asked
- ✓ Select they question type for each question type and specify the wording
- ✓ Design the question sequence and overall questionnaire layout

Determine the Questions to be Asked

Obviously, your questions should relate directly to the aim of the survey and to the specific information that you will require.

Decide on layout and sequence

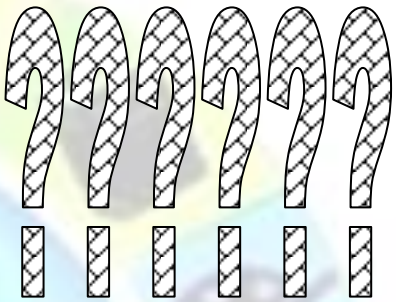
- ✓ Do not clutter up the form with unnecessary headings and numbers.
- ✓ Include the contact and return information on the questionnaire, irrespective of whether addressed return envelopes are provided, these can easily become separated.
- ✓ Identify individual questions for reference purposes.
- ✓ Be careful not to overfill the page.
- ✓ Avoid using lots of lines, borders and boxes since these can make the page look too 'dense'.
- ✓ Small fonts may put people off, especially people with bad eyesight.
- ✓ Use a good legible font.
- ✓ Make good use of italics and bold types, think of using italics consistently to give instructions.
- ✓ Consider using bold for the questions themselves or for headings. Symbol fonts may also be useful.
- ✓ Begin with questions that will raise interest.

- ✓ You should try to keep the flow through the questionnaire logical and very simple, i.e. avoid any complex branching.

Question types

Open-ended Questions: E.g. Do you think football hooliganism is caused by: (*tick if appropriate*)

Lack of discipline at home	
Players' behaviour on pitch	
Family breakdown	
Youth unemployment	
Poor schooling	
Violence on T.V.	
Other (please specify)	



Single vs. Multiple Response: E.g. What is your most usual means of travelling to college?

Bus	
Car	
Bike	

Rated Response: A popular approach in the social science is to use Likert scales such as the example below.

Please state how often you use the following: (*Please circle the numbers as appropriate)*

	Very often	Often	Occasionally	Never
Newspapers	1	2	3	4
Books	1	2	3	4
Periodicals	1	2	3	4

Wording of questions

When you compile a questionnaire, think carefully how you phrase the questions that you are going to ask the people out there. You have to phrase the questions in such a manner that the people who complete them must:

- ✓ Be able to understand the question
- ✓ Be able to answer the question
- ✓ Be willing to give you the information you need

You also have to ensure that the questions and the way you ask them cannot be constituted as biased in any way, e.g. biased based on race, gender, age, religion, culture, etc.

Formatting

Most survey forms begin with a brief statement of introduction to announce the survey, request participation, assure confidentiality (if appropriate), and indicate how to return the survey form. A letter of introduction is included with mailed survey forms although sometimes introductory letters are used in other settings as well.

The important part of the survey form usually includes the questions or statements to which respondents must react. In practice, most questionnaires place demographic questions (gender, age, racial group) first but some researchers argue that demographics should be placed last to avoid boring people with dull background questions at the beginning of the survey.

Next, questions that rely on the same sort of response mode (multiple choice, true and false) are grouped together. Each section should be preceded by instructions for completing the items. Some researchers recommend that questions that deal with the same issue should be grouped together. However, this ordering sometimes creates bias in response as people may try to respond in ways that are consistent with early statements on the survey. For example, suppose I ask you to evaluate the quality of teaching in statistics classes and at a later point ask you to state what you believe are the most important needs at your place of work. It is quite likely that you will mention the quality of teaching as being one of the important needs.

Although you can't avoid the effect of question order, you should attempt to estimate what the effect will be. This will allow you to interpret results in a meaningful fashion. If the order of questions seems an especially important issue, you might prepare more than one version of the questionnaire where each contains different ordering of questions.

The correct length for a questionnaire almost always poses a problem for researchers. In general, brief questionnaires are preferred to lengthy ones but sometimes you may have to prepare long questionnaires.

Formative assessment

In a group, draw up a list of at least six questions for your questionnaire

Run A Pilot Survey

Test the questionnaire on a small sample of your subjects first. If this is not possible at least test it on some colleagues or friends. The aim here is to detect any flaws in your questioning and correct these prior to the main survey.

Having done your pilot survey, you can make amendments that will help to maximise your response rate and minimise your error rate on answers.

Formative assessment

Use the other groups in the class as samples for your pilot survey. They must answer the questions drawn up in exercise 5 for your group.

Carry Out The Main Survey

The purpose of doing a pilot survey is to find out if you have to change anything in the questionnaires or in your population sample or even the aim of your survey.

If you are using fieldworkers, you have to ensure that they are well trained to minimise errors in the collecting of data.

- ✓ Errors when choosing respondents
- ✓ Interviewer dishonesty
- ✓ Misinterpreting or misreporting of information
- ✓ Non responses: where people are not at home or refuse to answer questions

For the process of actually doing the market research, you also have to

- ✓ Set deadlines: start on a specific day and end on a specific day
- ✓ Determine the number of questionnaires you want to complete by that day
- ✓ If you employ field workers, how many questionnaires every day and how many at the end of the period
- ✓ You have to put an administrative process in place: who is going to collect the completed questionnaires?
- ✓ You must also have a quality and cost control system in place to prevent dishonesty and prevent fieldworkers from charging too much and wasting too much time. You could, for example, pay per correctly completed questionnaire.

Formative assessment

In your group, look at the replies you received from the pilot survey. Is there anything you would change on the questionnaire? Make the changes to your questionnaire and indicate the changes below.

Interviews

Questionnaires are handy but researchers sometimes find interview methods more useful for a couple of reasons. It is easy for many people to ignore a cold questionnaire but it may be difficult for them to ignore a live person who asks questions. Of course, the interviewer may arouse some suspicions but this is part of the job to involve respondents in the task. Interviews are conducted to increase the willingness of the respondent to participate and to obtain information that may be lost with a questionnaire.

The interviewer may record information (such as a respondent's manner and body language) that would be absent with the questionnaire method. Of course, there is some art involved in completing interview studies. So when you read studies that use the interview method, imagine the give and take involved and the pressures the researcher and respondent felt.

- ✓ The interviewer should be warm and professional.
- ✓ Establish rapport to make respondents feel at ease.
- ✓ Professional appearance is important

REPRESENT, ANALYSE AND INTERPRET DATA

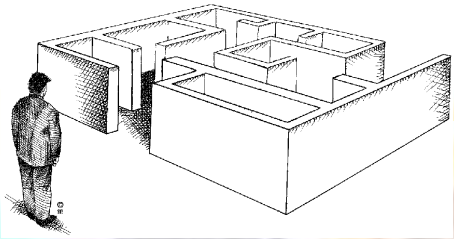
Outcome

Represent, analyse and interpret data using various techniques to investigate real-life and work problems

Assessment criteria

- Graphical representations and numerical summaries are consistent with the data, are clear and appropriate to the situation and target audience
- Different representations of aspects of the data are compared to take a position on the issue
- Calculations and the use of statistics are correct and appropriate to the problem
- Interpretations of statistics are justified and applied to answer questions about the problem
- New questions that arise from the modelling of the data are discussed

Analyse The Data



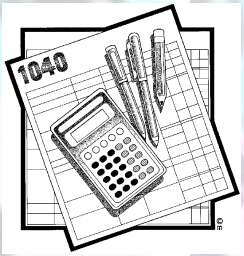
Now you have all the information, you have to do something with it: analyse the information. The only reason you went to all the trouble of conducting market research is that you want information, so that you can analyse the information and use it for the benefit of your new venture.

Once you have all the information, you have to organise it, summarise it and simplify the information so that you can make sense of it.

The easiest would be to prepare a document that lists all the questions. You then count all the answers and add the totals to the document. If we use the questions from a previous Formative Assessment:

Do you play computer games?	Yes	1450	No	550

Now you can calculate a percentage of the sample: who plays computer games and therefore might be interested in buying a new game.



- ✓ Total questionnaires received: 2000
- ✓ Total Yes 1450 percentage of sample: 72%
- ✓ Total No 550 percentage of sample 18%

This means that, of the people who took part in the market research, 72% do play computer games.

Formative assessment

Analyse the responses for the following questions:

Why do you use a taxi to and from work	Cheap	1631
	Fast	1091
	Safe	312
	Convenient	1849

If you sent out 2000 questionnaires, what percentage of the sample use taxis because they are cheap, what percentage use taxis because they are fast, what percentage use taxis because they are safe and what percentage use taxis because they are convenient?

Cheap	
Fast	
Safe	
Convenient	

Which taxi route do you use every day?	Route A	755
	Route B	830
	Route C	415

What percentage uses Route A, Route B and Route C?

Route A	
Route B	
Route C	

You would do this for all the questions.

On the basis of the above information, you can now make a decision as to whether there is actually place in the market for a new taxi service, or whether your improved service will satisfy a need with the customers.

Formative assessment

In a group, analyse the responses to the questions that you received from the other group during your pilot survey. Write down your conclusions.

Determining Trends Using Statistics

We determine and identify trends in regarding issues that affect our society, such as

- ✓ crime and health;
- ✓ relevant characteristics of targets such as age, range, gender, socio-economic , cultural belief and performance;
- ✓ and the attitudes or opinions of people on issues by doing research as indicated and then by interpreting this information statistically.

Once we have the data we look at common grounds and averages to determine these trends.

Mean, median, and mode

Mean, median, and mode are three kinds of "averages". There are many "averages" in statistics, but these are, I think, the three most common, and are certainly the three you are most likely to encounter in your pre-statistics courses, if the topic comes up at all.

- ✓ The "mean" is the "average" you're used to, where you add up all the numbers and then divide by the number of numbers.
- ✓ The "median" is the "middle" value in the list of numbers. To find the median, your numbers have to be listed in numerical order, so you may have to rewrite your list first.
- ✓ The "mode" is the value that occurs most often. If no number is repeated, then there is no mode for the list.

Describing data

Tables organize data and graphs present a clear overall picture of the distribution and spread of the data. However, I want to be able to describe the data in specific detail. I want to say that the centre of the data is 'here' and it has a spread 'this large'.

This section shows us how to calculate the centre of any distribution of values and to tell how spread the values are from the centre.

In general, the set of numbers used in this section is kept to a minimum. The techniques described, however, work on any number of items. A smaller set of numbers allows the principles to be demonstrated by hand or with a calculator. A large set of numbers would just become an exercise in concentration and patience, unless of course, you are using a computer to do the work for you.

Measuring centre or average

I have a lot of data that I've collected and I would like to represent that data by one number: its centre. I want to know the average value of my data.

It turns out that if there isn't just one way of measuring the centre of my data but there are many ways to do so. All are correct and all are used for different reasons. But the most commonly used methods are the mean, the median and the mode. In statistics-talk these are three 'measurements of central tendency'.

Calculating centres and averages

When calculating centres and averages we are trying to determine where the middle of the data are. There are three main methods used for calculating the centre of data: mean or average, mode and median.

Calculating the mean or average

The arithmetic mean is the average we all know and use: add up all the numbers and divide by their count (how many there are).

The mean of the following 5 numbers: 8, 3, 12, 5 and 10 is:

$$\bar{x} = \frac{8+3+12+5+10}{5} = 7.6$$

The '5' under the line (in the denominator) is the number of items or the count of items and has nothing to do with the value '5' above the line (in the numerator).

If I had 50 numbers I would add all 50 numbers together and divide by 50.

When do we use it? Suppose, for example, that at a party there are ten people aged 14, 15, 16, 14, 15, 16, 16, 15, 60, and 65 respectively. The mean of these ages is 24.6 which are not at all typical of the people at the party. A better statistic would be a median.

Calculating the mode

The mode is simply the number that occurs most frequently.

The mode of this sequence of numbers (2, 2, 5, 7, 9, 9, 9, 10, 10, 11, 12 and 18) is 9. The number '9' occurs the most number of times. This set of numbers is call 'unimodal' because it has one mode. (The set of numbers used to calculate the mean has no mode because all numbers occur the same number of times.)

The following sequence of numbers (2, 3, 4, 4, 4, 5, 5, 7, 7, 7 and 9) has two modes and is called 'bimodal'.

If a set of numbers has more than two modes the set is called 'multimodal'.

Mode is used if you have the same number occurring so frequently in a set of data that it can be regarded as the typical item. Suppose, for example that members of a group are asked to contribute to a gift for another person, and the contributions are

R1, R1, R2, R100, R100, R0.50, R2, R1, R1, R100, R100, R100.

This set of data can be described well by saying that the majority of people each contributed R100.

The mode is useful when dealing with nominal data (grouped data) like eye colour or ordinal data (ordered data) like shoe sizes. The

Shoe size	Sold
5	5
5½	8
6	18
6½	22
7	25
7½	32
8	45
8½	56
9	67
9½	82
10	98
10½	115
11	100
11½	89
12	60
12½	46
13	30
13½	19

mode is the value occurring most frequently. The shoe salesperson might have the following range of shoes with the associated number of sales during one month as shown in

The mode is the most useful measure of the centre of this data: size 10½. The salesperson now has an idea of the most common shoe size. To look at it another way: if I averaged the shoe sizes and told the salesperson the average shoe size you sold this month is 9¼, do you think this 'statistic' is meaningful? I don't!

As an exercise in visualizing a table of data, imagine the previous data plotted as a distribution or histogram. Can you visualize the distribution? Frequency or relative frequency distributions are most commonly displayed in histograms. The ages that were displayed in the pyramid diagram are two histograms placed back to back

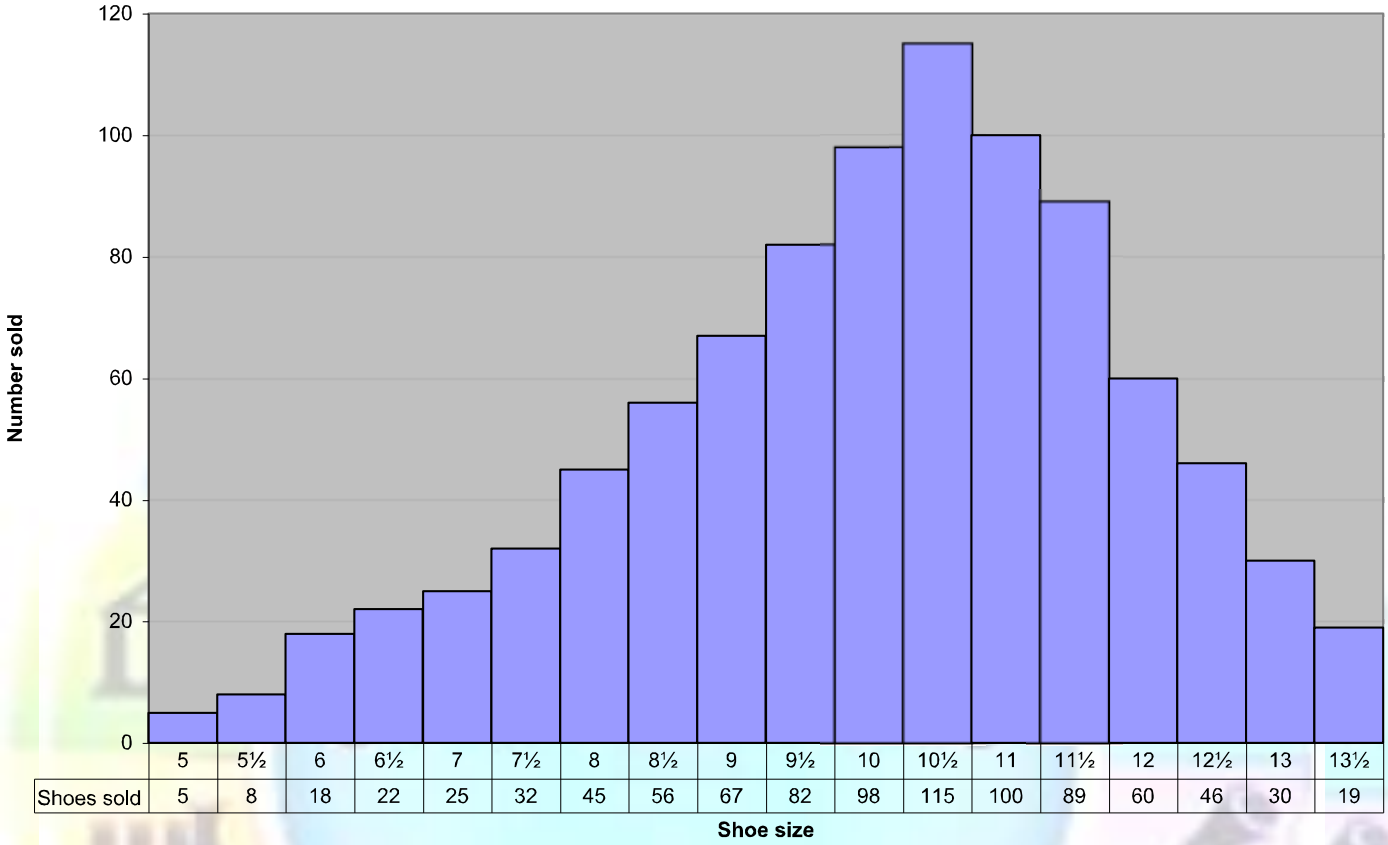
SHOE SIZES AND QUANTITY SOLD

Histograms look like bar charts but they are different.

- ✓ Histograms may only be construction of data on an interval/ratio scale (age, length, height). Divide the range of data into classes of equal width. In the example dealing with ages, groups were formed of people with ages in five-year intervals (0-4, 5-9, 10-14). You must ensure that the classes are precisely specified and no data can fall into more than one class.
- ✓ Count the number of observations in each class. These counts are the frequencies of the classes.
- ✓ Draw the histogram remembering to keep the data scale horizontal and the frequency scale vertical. Each bar represents a class. The base of the bar covers the class and the bar height is the class frequency. A histogram is drawn with no horizontal space between the bars unless a class is empty (has zero height).
- ✓ Bars in a histogram are vertical and the base scale is marked off in equal units. Bars in bar charts may be vertical or horizontal and there is no base scale. In other words, I may use any type of scale in a bar chart although nominal or ordinal scales are the most commonly used.
- ✓ The widths of the bars in a histogram have meaning. The base of each bar covers a class (interval) of values and the height represents the class frequency. The widths of bar charts have no meaning but they should all be the same so that they do not confuse or deceive you.
- ✓ The bars in a histogram touch each other unless there is a class with no entries. The bars cover an entire range of values. Even when the values of a variable have gaps between them, we extend the bases to meet halfway.

Remember that our eyes and brains respond to the area of the bars in a histogram. Therefore, all the widths must be equal, just like in a bar chart and for the same reason: so that they do not confuse or deceive you.

Histogram of shoe sizes sold during one month



	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12	12½	13	13½
Shoes sold	5	8	18	22	25	32	45	56	67	82	98	115	100	89	60	46	30	19

Calculating the median

The median is the middle value of the set of sorted values. Note that the data must be sorted before you can calculate the median by hand. A calculator or computer automatically sorts the set of numbers before calculating the mean.

When we calculate the median we are not concerned with the values of the set of data but rather the position of each number in the sorted order. This is important so pay attention.

There are two separate problems with calculating the median: an odd number of data values and an even number of data values.

For an odd number of data values (3, 4, 4, 5, 6, 8, 8, 8 and 10) select the middle value: 6. If you look at this data set you will see that there are four values below (smaller value) and four values above (larger value) 6. Therefore, 6 is the median.

For an even number of data values (5, 5, 7, 9, 11, 12, 15 and 18) take the average of the middle two values. In this case the median is 10. That is ½(9 + 11) = 10. The values 9 and 11 are the middle two values because they each have three values below and three values above them. Just average the middle two numbers.

Formative assessment

Follow the steps as detailed

Find the mean, median, and mode for the following list of values: 13, 18, 13, 14, 13, 16, 14, 21, 13

mean = average

The ***mean*** is the usual average, so:

(13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13) ÷ 9 = 15

Note that the mean isn't a value from the original list. This is a common result. You should not assume that your mean will be one of your original numbers.

The ***median*** is the middle value, so I'll have to rewrite the list in order:

13, 13, 13, 13, 14, 14, 16, 18, 21

There are nine numbers in the list, so the middle one will be the (9 + 1) ÷ 2 = 10 ÷ 2 = 5th number:

13, 13, 13, 13, 14, 14, 16, 18, 21

median = middle

So the median is 14.

The ***mode*** is the number that is repeated more often than any other, so 13 is the mode.

mean: 15

median: 14

mode: 13

mode = most often

Find the mean, median, and mode for the following list of values: 1, 2, 4, 7

The mean is the usual average: (1 + 2 + 4 + 7) ÷ 4 = 14 ÷ 4 = 3.5

The median is the middle number. In this example, the numbers are already listed in numerical order, so I don't have to rewrite the list. But there is no "middle" number, because there are an even number of numbers. In this case, the median is the mean (the usual average) of the middle two values: (2 + 4) ÷ 2 = 6 ÷ 2 = 3

The mode is the number that is repeated most often, but all the numbers appear only once. Then there is no mode.

mean: 3.5

median: 3

mode: none

The list values were whole numbers, but the mean was a decimal value. Getting a decimal value for the mean (or for the median, if you have an even number of data points) is perfectly okay; don't round your answers to try to match the format of the other numbers.

Find the mean, median, and mode for the following list of values: 8, 9, 10, 10, 10, 11, 11, 11, 12, 13

The mean is the usual average: (8 + 9 + 10 + 10 + 10 + 11 + 11 + 11 + 12 + 13) ÷ 10 = 105 ÷ 10 = 10.5

The median is the middle value. In a list of ten values, that will be the (10 + 1) ÷ 2 = 5.5th value; that is, I'll need to average the fifth and sixth numbers to find the median: (10 + 11) ÷ 2 = 21 ÷ 2 = 10.5

The mode is the number repeated most often. This list has two values that are repeated three times.

About the only hard part of finding the mean, median, and mode is keeping straight which "average" is which. Just remember the following:

- ✓ mean: regular meaning
- ✓ median: middle
- ✓ mode: most often

Frequency distribution and range

When you are drawing a chart or graph, you will need the frequency distribution and range of the information in order for the graph or chart to make sense.

The range is defined as the difference between the largest and smallest values in the data set.

range = largest - smallest

The range is one measure of the spread of a set of data. If the range is very large we may expect the values in the data set to be spread widely.

Definitions

- ✓ ***Frequency distribution***: where you arrange (distribute) data in some kind of order. A frequency distribution tells you how often certain numbers or values occur.
- ✓ ***Population***: The objects we are busy investigating (in the example on the next page, the learners in this class)
- ✓ ***Range***: The difference between the lowest and highest items in a set of data is called the range of the data set.

Formative assessment

Use the names of learners in your class and the number of children they have to complete the table on the following page:

Names of Learners in your class	Number of Children					
	1	2	3	4	5	6

1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						

We would like to indicate how many learners (number of cases) fall within the class intervals. The class intervals range from 0 – 1children, 2 – 3 children etc.

Class Interval	Tally (number of learners)	Number of children
0 - 1		
2 - 3		
4 - 5		
6 and more		
TOTAL:		

What is the range of the data set?

Graphical Representation Of Data

The aim of statistics is to provide insight by means of numbers. In order to achieve this we must collect numbers that are valid in the sense of being both correct and relevant to the issue at hand.

Once you have analysed the information, you will want to present it in such a way that everyone understands the information.

Data are presented in tables and graphs. This section covers some simple ways to represent data in graphs and to begin seeing how these numbers are distributed. The 'distribution' of a variable simply describes the values the variable takes on and how often each value occurs.

We want to use our knowledge of statistics to communicate facts and to support decisions. Data, like words, need to be organized in order to tell us anything useful. Tables of data are usually very large and I believe the best place for them is in a computer storage device or in an archive somewhere. I may need to get at these data for some or other reason but I don't want their volume to confuse me. I like simple things that I can understand.

Displaying data

I am going to start of with a table of data, taken from the 2001 census and published by Stats SA.

Language	Male	Female	Total
Afrikaans	2,900,214	3,083,212	5,983,426
English	1,772,483	1,900,720	3,673,203
IsiNdebele	342,366	369,455	711,821
IsiXhosa	3,726,376	4,180,777	7,907,153
IsiZulu	5,045,450	5,631,855	10,677,305
Sepedi	1,987,170	2,221,810	4,208,980
Sesotho	1,704,071	1,851,115	3,555,186
Setswana	1,774,785	1,902,231	3,677,016
SiSwati	571,429	623,002	1,194,431
Tshivenda	482,134	539,623	1,021,757
Xitsonga	1,001,446	990,761	1,992,207
Other	126,117	91,175	217,292
Total	21,434,041	23,385,736	44,819,777

FIRST HOME LANGUAGE BY GENDER: SOUTH AFRICA, 2001

This table lists the languages along with the number of people, grouped by gender, who use this language as their home language. Look at the table. It contains a few very important details.

Firstly, a table is made up of rows and columns. The rows extend horizontally across the page and the columns extend vertically down the page. Secondly, the table has a caption that describes the contents of the table. In this case the caption is below the table while in other cases the caption may be above the table. This table has a total column as its last row that may or may not appear in other tables. Thirdly, each column is labelled (Language, Male, Female and Total) so that you know what is in each column of data.

A table is pretty simple. It lists data in rows and columns and you can find the information you are looking for by going to either the row or columns of interest and looking either across or down. The intersection of the row and column (where they cross) is the data you require. In this example, if I want to find out how many people reported that they speak a home language that is not one of the official nine languages, I would look in the row called 'Other' and read the number to its right in the 'Total' column: 217,292.

Frequency tables

One of the first things to do when organizing a set of data is usually to count how often each value occurs. Stats SA kindly did this for us and presented the data in the form of a table

Because rates or proportions are often more useful than totals, we calculate these and display them per the table on the next page.

The technique used to create a table of proportions or ratios is simple but can become a little tedious without a calculator or a computer program to do it for you. If you are doing this by hand, that is, actually dividing the numbers, you will probably make a few mistakes. So check your work again.

	Male	Female	Total
Afrikaans	0.14	0.13	0.13
English	0.08	0.08	0.08
IsiNdebele	0.02	0.02	0.02
IsiXhosa	0.17	0.18	0.18
IsiZulu	0.24	0.24	0.24
Sepedi	0.09	0.10	0.09
Sesotho	0.08	0.08	0.08
Setswana	0.08	0.08	0.08
SiSwati	0.03	0.03	0.03
Tshivenda	0.02	0.02	0.02
Xitsonga	0.05	0.04	0.04
Other	0.01	0.00	0.00

Total	1.00	1.00	1.00

The basic idea of creating a proportion of a ratio is to divide the individual value by the total value. In the case of our example, the totals are already given, so you probably won't have to calculate them again. Or should you? Ensuring your data is correct is called 'internal consistency'.

Formative assessment

Do the calculations that follow

I'll show you the calculations for the first three languages for 'Male', 'Female' and 'Total' in order to show you how easy it is to do.

Male:

$$\frac{2,900,214}{21,434,041} = 0.1353 \approx 0.14 \text{ or } 14\% \text{ for Afrikaans, male}$$

$$\frac{1,772,483}{21,434,041} = 0.0827 \approx 0.08 \text{ or } 8\% \text{ for English, male}$$

$$\frac{342,366}{21,434,041} = 0.0160 \approx 0.02 \text{ or } 2\% \text{ for IsiNdebele, male}$$

Female:

$$\frac{3,083,212}{23,385,736} = 0.1318 \approx 0.13 \text{ or } 13\% \text{ Afrikaans, female}$$

$$\frac{1,900,720}{3,385,736} = 0.0813 \approx 0.08 \text{ or } 8\% \text{ English, female}$$

$$\frac{369,455}{23,385,736} = 0.0158 \approx 0.02 \text{ or } 2\% \text{ IsiNdebele, female}$$

Total:

$$\frac{5,983,426}{44,819,777} = 0.1335 \approx 0.13 \text{ or } 13\% \text{ for Afrikaans, total}$$

$$\frac{3,673,203}{44,819,777} = 0.0820 \approx 0.08 \text{ or } 8\% \text{ for English, total}$$

$$\frac{711,821}{44,819,777} = 0.0159 \approx 0.02 \text{ or } 2\% \text{ for IsiNdebele, total}$$

If you check the results shown in the table from Stats SA, by summing each column you will find that the sums are 1.01, 1.00 and 0.99 for the 'Male', 'Female' and 'Total' columns. What happened? The arithmetic is correct but when I rounded the fractions to two decimal places, a

little precision was lost. These errors are called 'round-off errors' or 'rounding errors' and they will be with us any time we round numbers. You'll get used to it!

- ✓ The 'frequency' of any value of a variable is the number of times that value occurs in the data. A frequency is a count.
- ✓ The 'relative frequency' of any value is the proportion or fraction or per cent of all observations that have that value.

In the example above, the total number of South Africans who use Afrikaans as a first language is 5,983,426. This is a frequency because it's a count of something: first language Afrikaans speakers.

The relative frequency is usually expressed in decimal form and in this case is 0.13. However, we could just as well express this value as a percentage (13%). Remember that 1% is $\frac{1}{100}$ or 0.01. A number in decimal form can be changed to a percentage by moving the decimal point two places to the right. (This is the same as multiplying the decimal form by 100 and putting a per cent sign, %, behind the result. 'Per cent' means 'by 100' or 'per 100',)

Frequencies and relative frequencies are a very common way of summarizing data when a nominal scale is used (gender, responses on questionnaires, eye colour).

In fact, it is such a handy way of summarizing facts that it is often used with an interval/ratio scale. In this case, we artificially group items and then count how many items fall into each group.

In this example, again taken from the census figures of Stats SA, we are looking at the age of a person. Age is measured on an interval/ratio and is a continuous value.

The table below shows the actual values of the age of all South Africans grouped into five years age groups. The table also displays the percentage of each age group. Note that the column totals do not sum to 100%. I only used three decimal figures to calculate the results in order to demonstrate the potential problems with round-off errors.

Age	Male	%	Female	%	Total	%
0-4	2,223,731	10.3%	2,226,085	9.5%	4,449,816	9.9%
5-9	2,425,804	11.3%	2,427,751	10.3%	4,853,555	10.8%
10-14	2,518,956	11.7%	2,542,961	10.8%	5,061,917	11.2%
15-19	2,453,079	11.4%	2,528,642	10.8%	4,981,721	11.1%
20-24	2,099,293	9.7%	2,195,230	9.3%	4,294,523	9.5%
25-29	1,899,124	8.8%	2,035,814	8.7%	3,934,938	8.7%
30-34	1,594,488	7.4%	1,746,412	7.4%	3,340,900	7.4%
35-39	1,441,507	6.7%	1,630,264	6.9%	3,071,771	6.8%
40-44	1,233,632	5.7%	1,385,832	5.9%	2,619,464	5.8%
45-49	967,604	4.5%	1,119,776	4.7%	2,087,380	4.6%
50-54	769,499	3.5%	868,521	3.7%	1,638,020	3.6%

55-59	552,323	2.5%	652,943	2.7%	1,205,266	2.6%
60-64	444,510	2.0%	620,784	2.6%	1,065,294	2.3%
65-69	304,763	1.4%	483,164	2.0%	787,927	1.7%
70-74	232,547	1.0%	398,922	1.7%	631,469	1.4%
75-79	136,436	0.6%	231,101	0.9%	367,537	0.8%
80-84	90,835	0.4%	180,111	0.7%	270,946	0.6%
85+	45,907	0.2%	111,425	0.4%	157,332	0.3%
Total	21,434,038	99.1%	23,385,738	99.0%	44,819,777	99.1%

AGE DISTRIBUTION AND RATIO IN FIVE-YEAR INTERVALS BY GENDER: SOUTH AFRICA 2001

Graphing data

Graphs, like tables, should be clearly labelled to show the variables that are being presented and the units being used. There are three things to remember when putting data in a graph:

- ✓ Make your data stand out
- ✓ Avoid clutter on the graph
- ✓ Use visual perception to get the facts to others.

An excellent example of a graph is shown in one the next page. This graph is the one Stats SA distributed with the age information in the previous example.

The impact of this graph along with its clarity of presentation is striking. This type of graph is called a pyramid or butterfly graph because of its shape and it belongs in the category of bar charts.

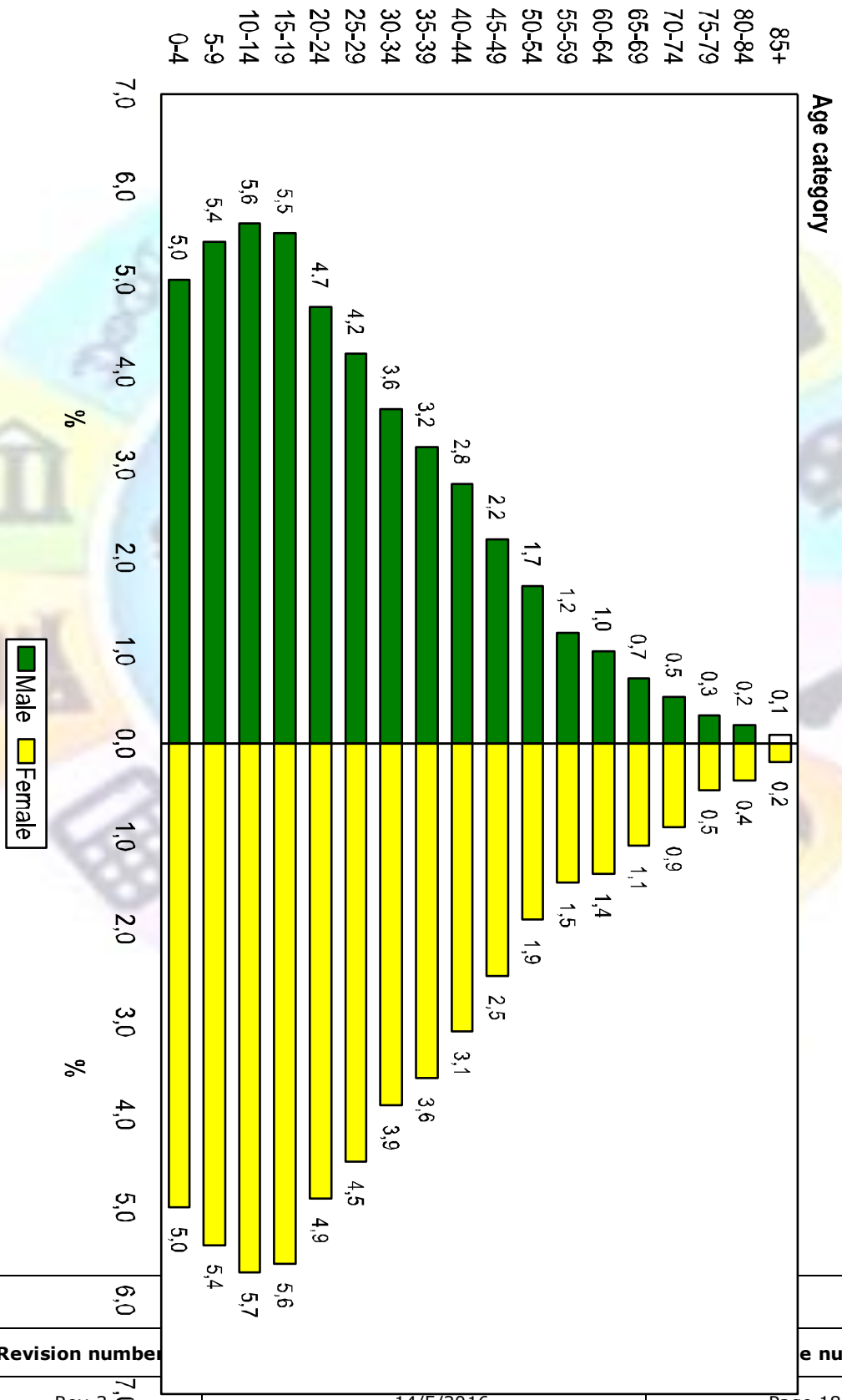
The vertical axis displays the age categories and the horizontal axis displays the percentage of each age group according to male and female. The data for the male groups is in green and is shown on the left while that for the female groups is shown on the right. This type of graph allows you to compare two groups of items at the same time.

I'm giving a bit of interpretation of this graph so that you can get the feel of looking at a picture and still see data. The very top of the graph shows that there are very few of us in the 85+ year group. This makes sense because as the population ages its population declines. In other words, the further the green or yellow lines are from the centre the more people there are in that age group. Conversely, the closer the green or yellow lines are to the centre the fewer people there are in that age group.

In addition, the female population slightly outnumbers the male population in every category with the exception of the bottom two where they are approximately equal. This makes sense from two viewpoints. Firstly, the female population is slightly greater than the male population (52% to 48%, respectively). Secondly, females tend to live about five years longer than males. This may also be seen in the top end of the graph (above the 35-39 year group) where the male to female ration starts declining. At the 70-74 year groups and up the number of females is approximately double the number of males.

.

AGE DISTRIBUTION OF MALES AND FEMALES IN THE TOTAL POPULATION, CENSUS 2001



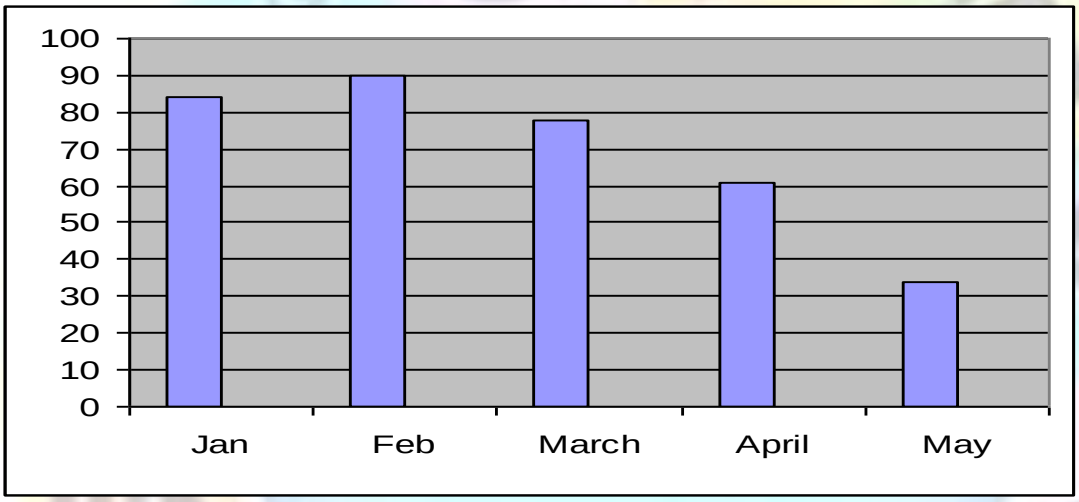
Bar graphs

Bar graphs compare measurements at intervals, the bars run horizontally. Column charts compare measurements at intervals and provide a view of data at a specific time. The bars run vertically

The example below shows a column chart indicating how many ice creams were sold from January to May.

If you use a bar chart, the bars will run horizontally and not vertically as with a column chart.

NUMBER OF ICE CREAMS SOLD



In a bar chart, the heights of the bars are important.

When you draw a bar graph, state clearly what you are representing on the two axes. This means that you have to label the axes. Also insert it on the graph above.

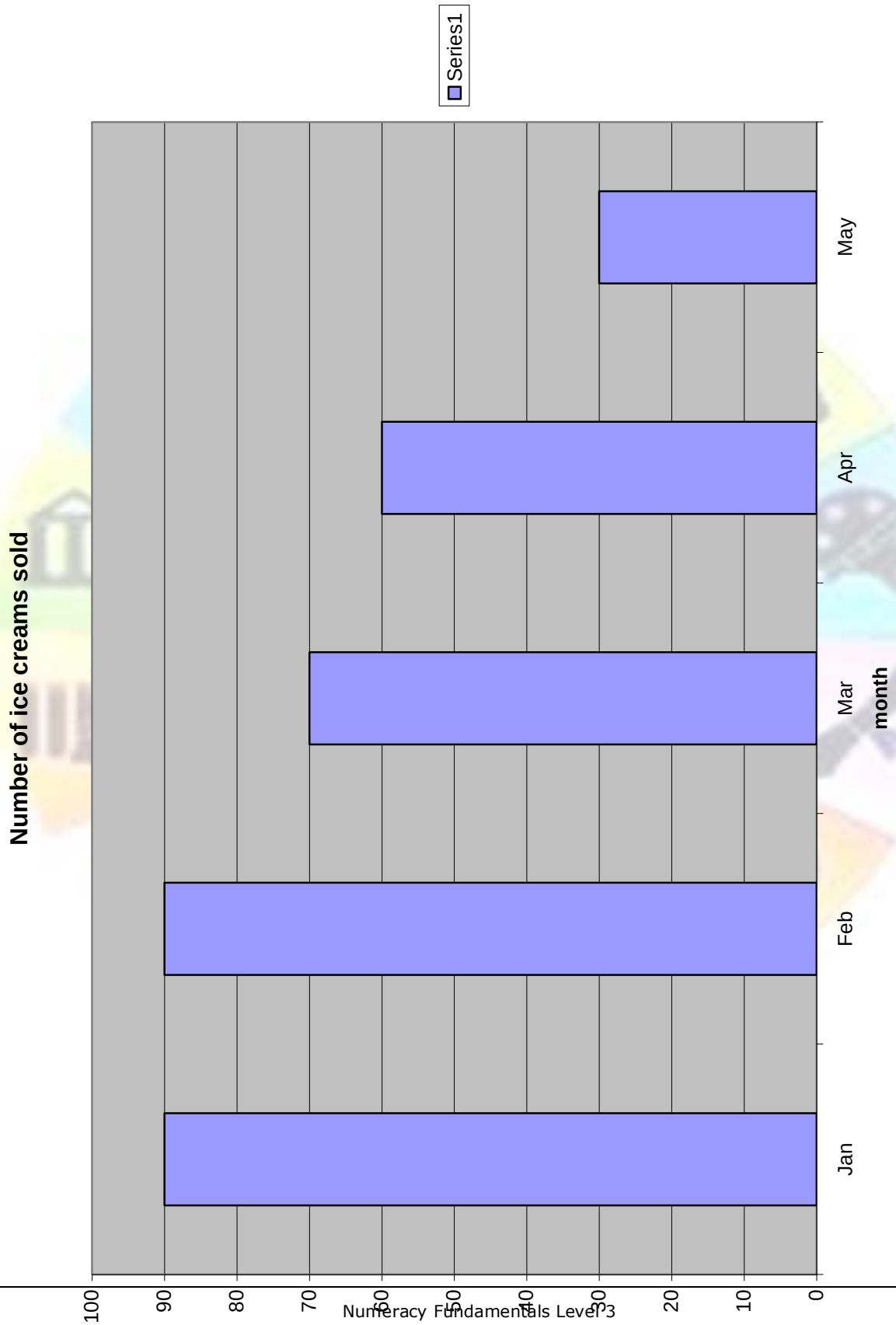
- ✓ Draw the axes at right angles to each other
- ✓ Choose a scale for the vertical axis and write in the units.
- ✓ Use a ruler to help you read off the height of a bar.

Formative assessment

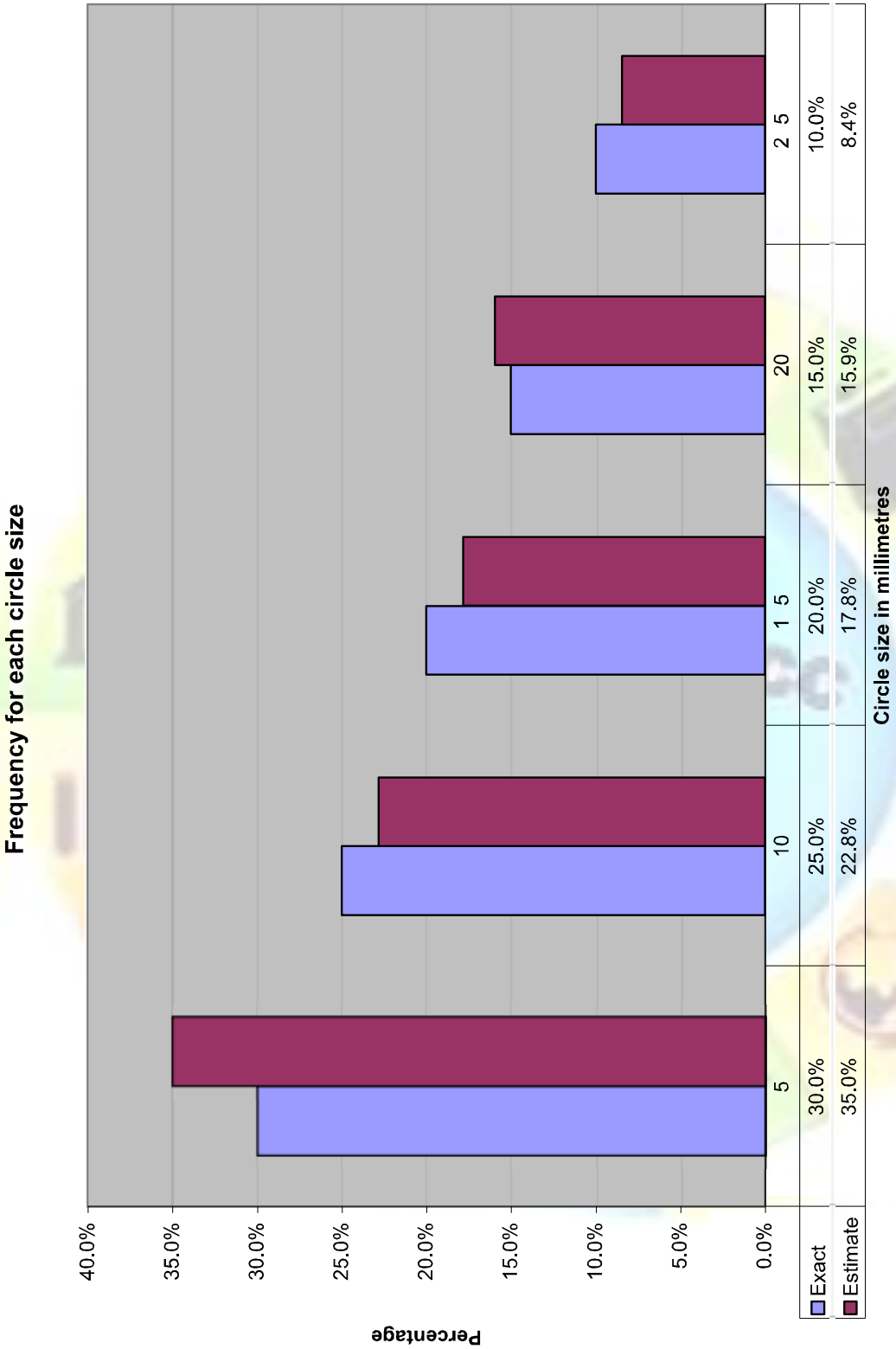
Study the chart on the next page and answer the questions below:

How many ice creams were sold during January?	
During which month were the most ice creams sold?	

In which month were 70 ice creams sold?	
What was the total number of ice creams sold for the period?	
What is the range of the data set?	

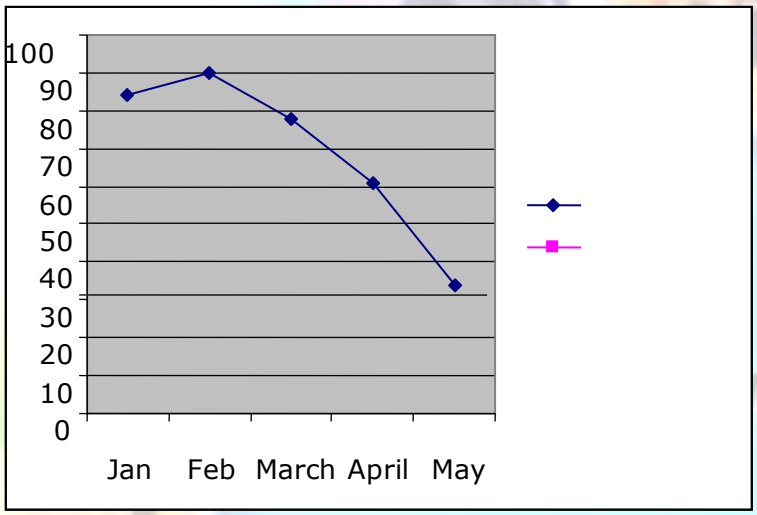

Series1
Number of ice creams sold
100
90
80
70
60
50
40
30
20
10
0
Jan
Feb
Mar
Apr
May
month
Number sold

The bars of the chart may be vertical or horizontal. They may touch each other as shown in the figure or they may overlap each other or be separated from each other. But be careful when creating your own bar chart! Each bar must have the same width because our eyes and mind respond to the area of the bars. When the bars have the same width and a height that varies with the variable then the area (height times width) also varies and our eyes and brains receive the correct impression.


Frequency for each circle size
Percentage
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
Exact
Estimate
5
10
1 5
20
2 5
30.0%
25.0%
20.0%
15.0%
10.0%
35.0%
22.8%
17.8%
15.9%
8.4%
Circle size in millimetres

Line Graphs or Curves

Show the changes in data or trends over a given period of time. They are used to emphasize rather than compare. We use a dot to show the height of each bar. If we join the dots, we get a line graph.



Formative assessment

The example above is a column chart indicating how many ice creams were sold from January to May. Study the column chart and answer the questions below

What tendencies do you pick up from this graph?	
When would be a good time to start a new ice cream business?	

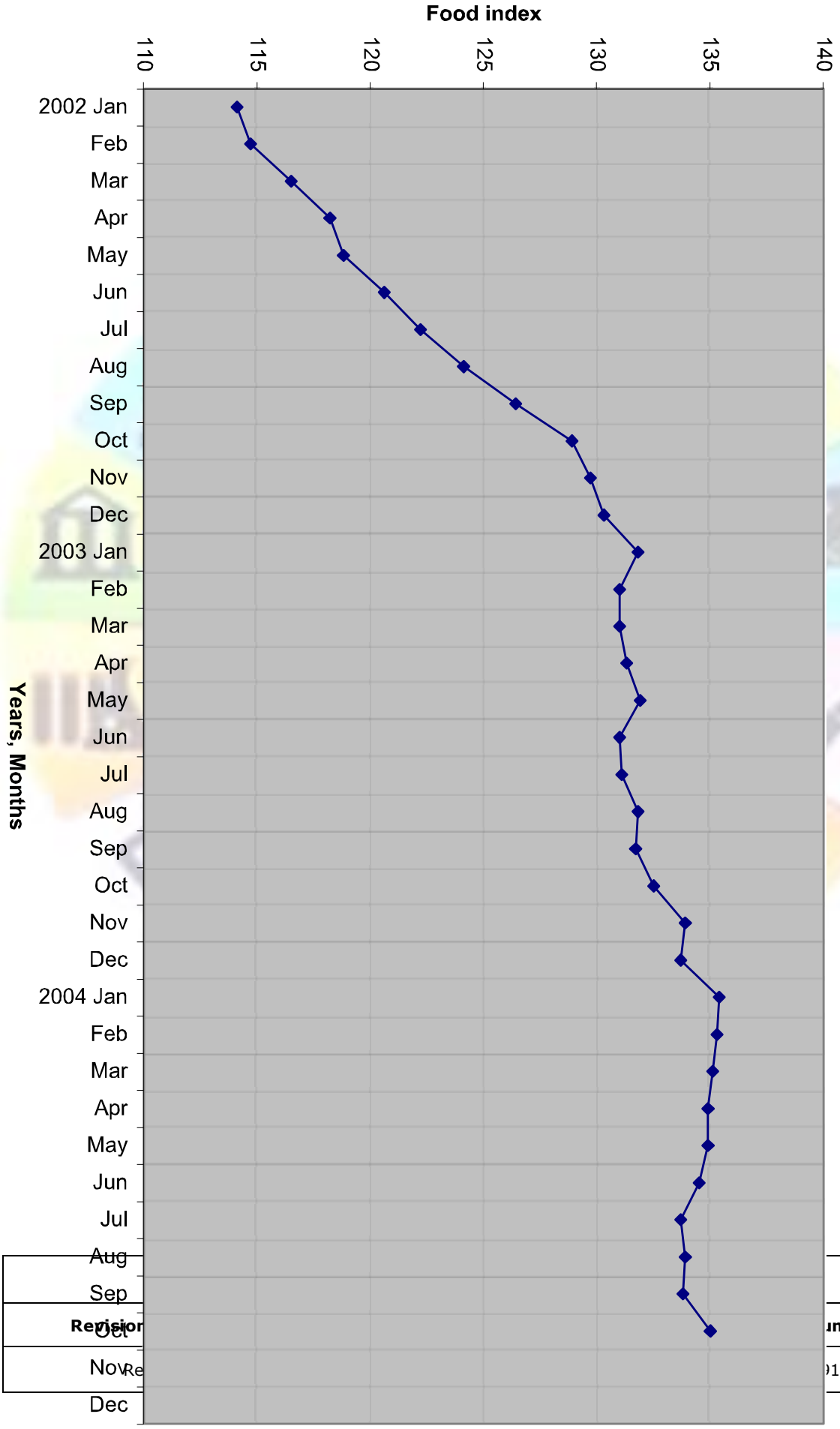
Line graphs show the behaviour of a variable over time. Time is placed on the horizontal axis and the variable being plotted is shown on the vertical axis.

A good example of a time based line graph is the food index from Stats SA. You will find this chart on the page following the ice cream chart.

Compiled by: Sakhisisizwe	Approved By: L.Levin

Food Index on a monthly basis for the historical metropolitan areas.

Base year: 2000 = 100



Revision	umber
Re	91

The graph makes it clear that the food index rose sharply between January 2002 and January 2003 after which its rise slowed somewhat.

As you can see in the graph, time (years and months) is displayed on the horizontal axis and the index value is displayed on the vertical axis. All time periods are of a fixed length and the length is the same for each month. I am pointing this out because I have seen graphs that try to distort facts by altering the graph in some respects.

A very handy feature with the line chart is that you may have several graphs on the same chart. This makes them a bit easier to compare. The line chart shows the Consumer Price Index for the historical metropolitan areas (CPI), the Consumer Price Index excluding interest rates on mortgage bonds (CPIX) and the Food Price Index (FPI). The FPI was shown on the last two figures.

In this graph we can see the same as before with the FPI, however, the CPI and CPIX are also shown in comparison. The interpretation of these index figures is not discussed here, however, you can see that more than one graph may be represented and comparisons may be made. The FPI represents the price of food, as you probably have already guessed. In comparison to the CPI and the CPIX note how sharply the FPI rose from about July 2002. If you remember, there was a big outcry concerning the cost of food and, in particular, basic food prices near the end of 2002. This graph shows you that rise.

Pie Charts

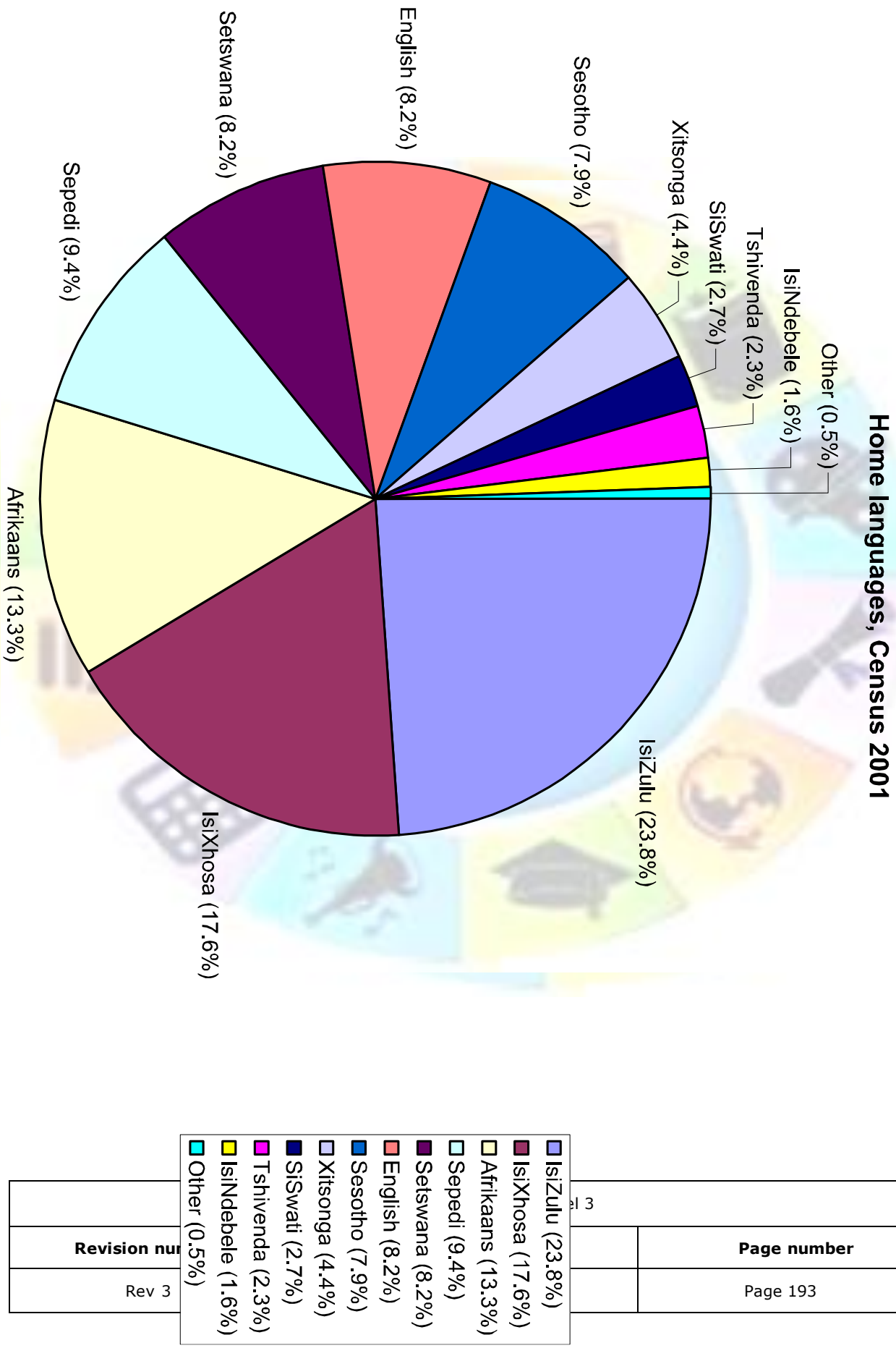
The pie chart shows how a whole is divided into parts. The home language distribution is shown on the next page.

The pie chart is a good option to choose when you want to show the relationships that parts have to the total. In this example, all the home languages are compared. They have been ordered from largest (IsiZulu) to the smallest (Other) and are displayed in this manner in the pie chart. The legend on the right as well as the labels and colouring of each section of the pie make the pie chart easy to understand.

Pie charts show us the parts that make up the whole but humans don't see angles as clearly as we see length.

For this reason, the pie chart is not a good choice to compare sizes of various parts with the whole. In addition, the divisions used in the example of the pie chart are causing the graph to become a bit crowded.

If I tried to do a pie chart of the age groups, it would probably look pretty messy! If not messy, it certainly would look crowded. I also think the 85+year group would be hard to see. There are alternatives to the pie chart!

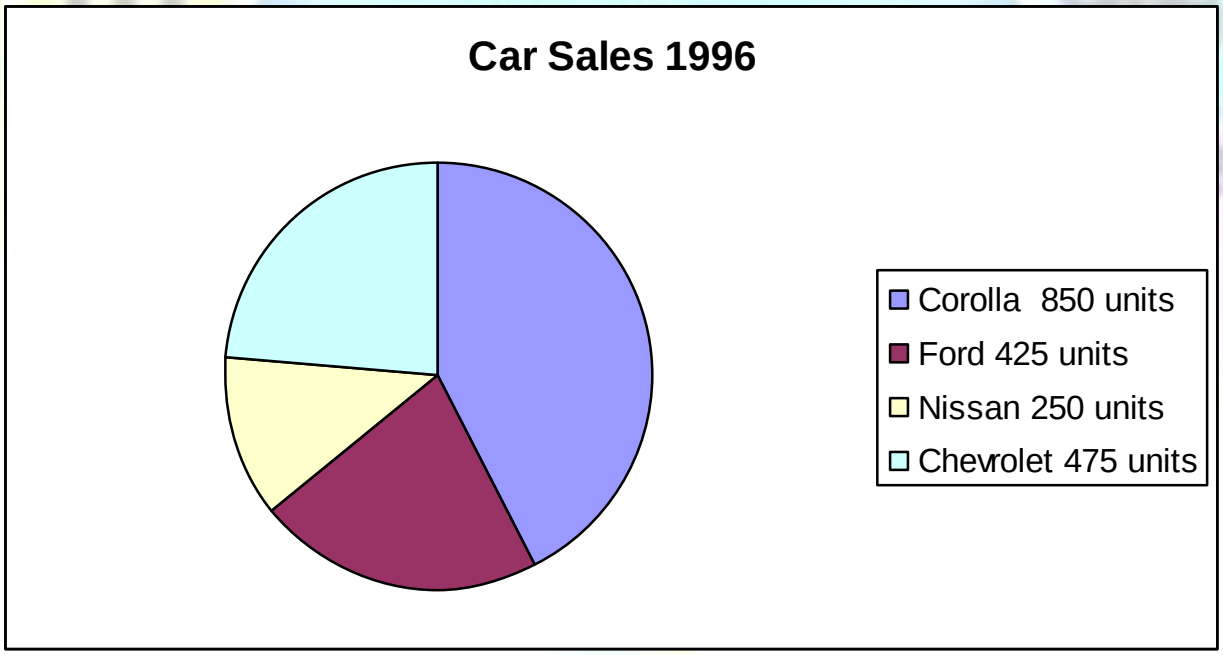

Home languages, Census 2001
Other (0.5%)
IsiNdebele (1.6%)
Tshivenda (2.3%)
SiSwati (2.7%)
Xitsonga (4.4%)
Sesotho (7.9%)
English (8.2%)
Setswana (8.2%)
Sepedi (9.4%)
Afrikaans (13.3%)
IsiXhosa (17.6%)
IsiZulu (23.8%)
IsiZulu (23.8%)
IsiXhosa (17.6%)
Afrikaans (13.3%)
Sepedi (9.4%)
Setswana (8.2%)
English (8.2%)
Sesotho (7.9%)
Xitsonga (4.4%)
SiSwati (2.7%)
Tshivenda (2.3%)
IsiNdebele (1.6%)
Other (0.5%)

A pie chart shows the breakdown of a total. A pie chart is a good way to show how a fixed number is divided. The whole circle (360°) represents the total number (or 100%) and we express each part as a fraction or percentage of the whole. A pie chart is constructed by converting the share of each component into a percentage of 360 degrees.

In the example below, a total of 2000 cars were sold in 1996 and the pie chart shows the breakdown of the 2000 cars: which manufacturer sold how many cars.

Formative assessment

Which company sold most cars?	
Which company sold the smallest percentage of cars?	
How many cars did Chevrolet sell during 1996?	
What is the range of the data set?	



Formative assessment

In a group draw a column chart or a bar chart for the following information. Use the grid below to help you.

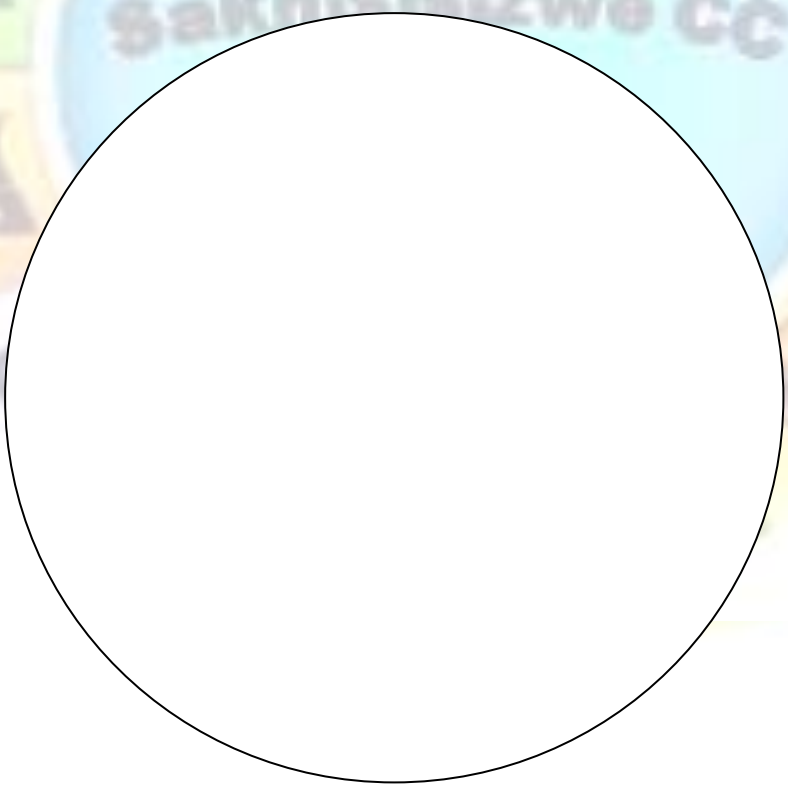
Why do you use a taxi to and from work	Cheap	1631
	Fast	1091
	Safe	312
	Convenient	1849

2200				
2000				
1800				
1600				
1400				
1200				
1000				
800				
600				

400				
200				
	Cheap	Fast	Safe	Conve-nient

In a group, draw a pie chart for the following information. A total of 2000 replies were received. Use the pie below to help you.

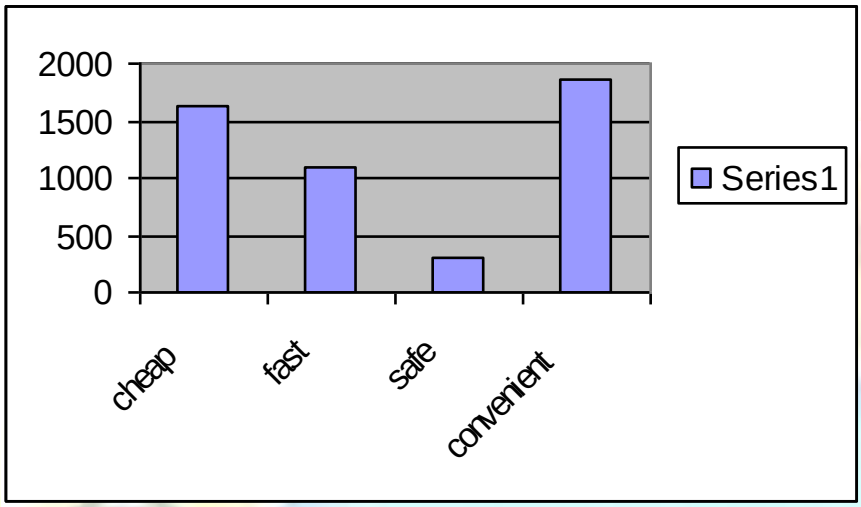
Which taxi route do you use every day?	Route A	755
	Route B	830
	Route C	415



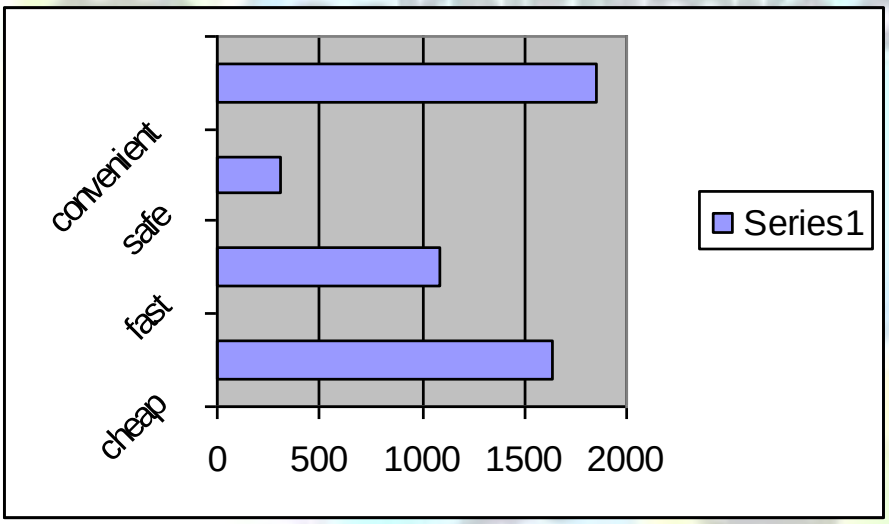
In a group, display the information you collected during the survey that was conducted in the class in the form of a chart.

Ideally, your charts should look as follows:

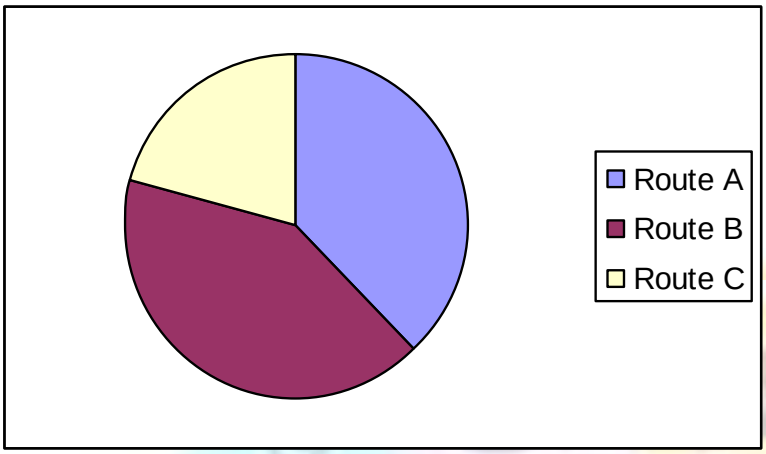
Your information as a column chart



Your information as a bar chart



Pie Chart



Comparing (Correlation) Of Data

When you do a survey, it is because you want to obtain certain information (data) and then you want to compare it to

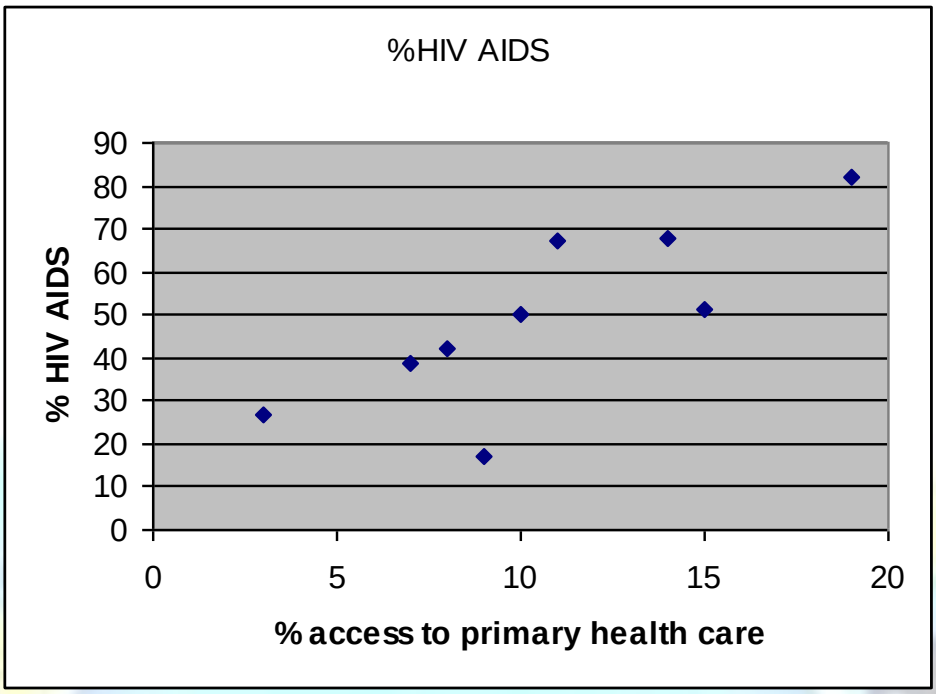
- ✓ what it should be,
- ✓ what the rest of the world does,
- ✓ what the ideal should be, etc.

Only once you have analysed and compared the data can you come to conclusions about the aim of your survey:

- ✓ how many ice creams were sold
- ✓ how many cars were sold
- ✓ how many employees become ill during the winter
- ✓ how many passengers do you transport on a specific route
- ✓ how much fuel does your bus or taxi use, etc.

Of course, all graphs and charts serve to compare information (data) and help you to reach a conclusion, but a correlation plot is a graph that shows the relationship between two sets of scores or values. In the example below, between the number of people who are HIV/AIDS positive and the access to primary health care

This correlation plot is shown as a scatter graph (a line chart without lines)



Formative assessment

In a group, refer to the correlation plot on the previous page and discuss the following statement, noting your conclusions:

What conclusions do you come to regarding the relationship between the occurrence of HIV AIDS and access to primary health care?

Summarising Data

The purpose of this section is to revise the calculation of averages by using mean, mode and median and also to give examples of when each average is appropriate.

There are often situations in which it is useful to summarise a whole set of data by describing it with a single number. The sum of a set of numbers can be used to summarise

data (e.g. the total mass of the pack of rugby forwards is an indication of the possible power of a rugby team).

The total is not always a useful summarizing number, however. Suppose we wish to compare the heights of men and women in a group. The totals cannot be used for the data given because there are different numbers of men and women.

Height of 10 year olds(m)	1.76	1.77	1.8	1.66	1.6	1.79	1.8		
Height of 9 year olds(m)	1.69	1.7	1.5	1.42	1.42	1.75	1.67	1.62	1.6

Although the total for the 9 year olds (14.37 m) is higher than the total for the10 year olds (11.12 m), when we consider individual heights it seems that 10 year olds are typically taller than 9 year olds.

What we actually need is a single number which is typical or representative of the heights of individual 9 year and 10 year olds. One such a number is the arithmetic mean or average.

Mean

Arithmetic mean = (sum of cases)/total number of cases

- ✓ Mean height of men = 11.12/7 = 1.59
- ✓ Mean height of women = 14.37/9 = 1.6

Median

Median is the middle value in a spread of values arranged in order from the lowest to the highest.

The median of an even number of items is the mean of the two middle items when the items are arranged according to size.

The median of an odd number of items is the middle item when the items are arranged according to size.

When do we use it? Suppose, for example, that at a party there are ten people aged 14, 15, 16, 14, 15, 16, 16, 15, 60, and 65 respectively. The mean of these ages is 24.6 which are not at all typical of the people at the party. A better statistic would be a median.

Mode

When numbers occur frequently in a set of data, the number occurring most frequently is the ***mode.*** This is used if you have the same number occurring so frequently in a set of data that it can be regarded as the typical item. Suppose, for example that members of a group are asked to contribute to a gift for another person, and the contributions are

R1, R1, R2, R100, R100, R0.50, R2, R1, R1, R100, R100, R100.

This set of data can be described well by saying that the majority of people each contributed R100.

For each set of data (information) that you have collected, you will have to decide which of the three statistics (mean, mode, and median) will give you the best description of the data.

Formative assessment

In a group, do the following: In each case state which of the three statistics is not an appropriate description of the given data. Order the data and draw a histogram of the data to see how it is distributed. If it is evenly distributed, the mean is most probably the best summary. If not, consider the median. If there are many occurrences of the same value, consider using the mode.

5 7 2 3 8 1 5 2 6

6 2 9 0 3 2 0 2 1 3 1 0 2

21 30 14 5 16 24 17 3 29

Using centres and averages

For each set of data (information) that you have collected, you will have to decide which of the three statistics (mean, mode, and median) will give you the best description of the data.

The ***mode*** is useful when dealing with nominal data (grouped data) like eye colour or ordinal data (ordered data) like shoe sizes

We now know how to calculate the mean, mode and average but we must still learn how and when to use them. Although the mode is very important in many real life situations it is not used that often in statistical calculations. In many situations the mode is not useful at all because there is no mode. The median is used more frequently than the mode because it is able to describe the data set with more flexibility. The median is also easily understood. The mean, or average, is the most commonly used measure of the centre of a set of data because it is backed by statistical theory. Let's now see these different ways of measuring the centre of data in action.

Assume that I am the owner of a sporting body and I employ twelve sportsmen (or sportswomen). The annual salaries of each sportsman (sportswoman) are as follows:

- ✓ six receive R200,000
- ✓ four receive R400,000
- ✓ one receives R800,000
- ✓ the superstar receives R2,400,000
- ✓ What is their average salary?
- ✓ If I used the mode to calculate the average salary it would be R200,000 because this value occurs with the highest frequency (6 times).

- ✓ If I use the median I must first order the salaries as shown in the table below and then determine the middle value:

Order	Salary
1	R200,000
2	R200,000
3	R200,000
4	R200,000
5	R200,000
6	R200,000
7	R400,000
8	R400,000
9	R400,000
10	R400,000
11	R800,000
12	R2,400,000.

There are an even number of entries (12) so I must obtain the average of the middle two values (R200,000 and R400,000) and indicate that the median is R300,000.

If I use the average (mean) I must sum all the salaries and divide by the number of players which is R500,000 (R60,000,000 divided into 12 players).

Which is the best number to use for the average salary?

The mode indicates that the highest frequency and most players receive R200,000. This number just doesn't seem right to me even though half the players receive this salary and the other half receive more.

The average produces R500,000. No player receives this amount but then it is an average. However, 10 players receive less that R500,000 and only two receive more. Again, this just doesn't look right to me.

The median also produces a value that no player receives but at least half the players receive less than this amount and half receive more than this amount. This is the middle of the road average so I would agree with it. As a matter of fact, if the superstar was paid R24,000,000, this measurement of the centre (R300,000) would not change and I would still have half of the players being less and half being paid more. However, if I used the average (mean) I would find that the average salary would be R2,300,000!

The median is a good choice to use for the central value when the distributions are skewed to the right or to the left. Salaries are almost always skewed to the right (very few people obtain very high salaries while most of the workers receive salaries that are on the lower end.) Note that the median value always has half the values on one side and half the values on the other side. There is always a middle value that exists or is the average of the two centre values. The values don't matter, only their positions when sorted matter.

When the distribution of values is more or less symmetrical and there are no outliers, then the average or mean is the best value to use for the centre of the data set.

Values that skew the distribution of values of the data affect the mean or average, sometimes dramatically. If I did pay my superstar R24,000,000 and advertised that my average salary bill is R2,300,000, the other 11 players would be at my door asking why their salary is so low!

PROBABILITY CONCEPTS IN LIFE

Outcome

Use random events to explore and apply, probability concepts in simple life and work related situations

Assessment criteria

- ✓ Data are gathered, organised, sorted and classified in a suitable manner for further processing and analysis.
- ✓ Experiments and simulations are chosen appropriately in terms of the situation to be investigated
- ✓ Probabilities are determined correctly
- ✓ Distinctions are correctly made between theoretical and experimental probabilities
- ✓ Predictions are based on validated experimental or theoretical probabilities.
- ✓ The outcomes of experiments and simulations are communicated clearly

Drawing Conclusions from Data

In order to draw conclusions from our data we have to understand it and to look at it to see what the data is telling us. In this section we are looking at simple probability theory and how to make confidence statements about our data.

Probability (Equal And Unequal Probabilities)

What is probability?

Even the rules of football agree that tossing a coin avoids favouritism. Probability is nothing more than tossing a coin!

In the Introduction, I said that we all have an innate tendency to understand and use statistics. And this is true. What is also true is that we humans ***do not have an innate ability to understand probability.*** We get confused and do silly things like gamble our money away or have another child to break the run of six boys (girls).

Because we have a memory we automatically think that chance event also have a memory. They don't! Whether we are looking at tossing a coin, rolling a die or two dice, playing the roulette wheel, having children or observing the offspring of animals, the outcome is purely random but totally predictable in the long run.

The ***'Law of Averages'*** doesn't exist with random events and we humans have difficulty understanding this. We think that after having tossed 10 heads in a row the 'Law of Averages' will turn up a tail next. It might, but the next toss still has a 50-50 chance of being either a head or a tail. A coin has no memory and it doesn't know that it turned up heads 10 times in a row.

When we gamble in games of chance, for example the roll of a dice, the spinning of a coin, roulette wheel, etc we consider the probability of winning: in the case of dice 1 out of 6 or 2 out of 12, flipping a coin (heads or tails) 1 out of 2 and, in the case of roulette, 1 out of 75 or higher. These probabilities are called "a priori" probabilities.

Experiment and observe

Most probabilities cannot be estimated by deduction.
Probabilities must be estimated only after experimentation and observation.

This is also true of future possibilities of the outcome of events.

It is possible to explain the outcome of a future event by observing its frequency of occurrence in the past. This data is then projected into the future.

You know that if you toss a coin enough times you will obtain about 50% heads and 50% tails. (Usually there is a little more metal on the 'head' side of a coin and it turns up slightly more frequently than tails.)

But how do we work out probabilities? We ***observe***. Only by observation can we be reasonably sure of the approximate value of the probability of an outcome.

If you have the time, try tossing a coin several hundred times and record the outcomes. You won't get exactly 50% head and 50% tails but you will see that 50% is the value you are heading towards.

Now take the same coin but instead of tossing it, hold it on end on a hard table surface with the index finger of one hand and snap it with the index finger of the other hand. The coin will spin for a while before falling with either heads or tails showing. Do not count the spins that fall off the table or the spins that bump into anything else. What do you expect to find: 50% heads and 50% tails?

A long series of trials reveals that the probability of a spinning coin is not $\frac{1}{2}$ ***.***

When we estimate the probability of our sport team winning on Saturday, we use deductive reasoning to make a bet on the outcome of the game. ***We sit down and work out what the chances are that our team will win, based on what they did in the past and which team members are playing, etc***. Such probability statements are referred to as ***statistical*** or ***inductive*** statements. These estimates are based on underlying assumptions:

- ✓ That enough past events have been observed.
- ✓ That casual influences will be unchanged and remain as they were in the past.

Definition

$$\text{Probability} = \frac{\text{number of possible outcomes to satisfy a specific condition}}{\text{total number of possible outcomes}}$$

When throwing the dice, the probability of throwing a 4 is 1/6 (one out of six). However, it could happen that the 4 is thrown more than once, let's say it was thrown twice (2/6). This fraction is then called the ***success fraction***.

The success fraction for an event is not necessarily equal to the probability of an event. However, if you repeat the experiment often and the results are combined, the success fraction for an event tends to draw closer to the probability of the event. The data also becomes less biased.

Odds on versus probability

Probability originated in gambling and in gambling the chance of an event is often stated in terms of odds rather than probability. You are rolling two dice and want to get a 7 but you have heard that the odds against this happening is 5 to 1.

The probability of a 7 is $\frac{6}{36} = \frac{1}{6} = 0.1667$ because there are 6 ways you may roll a 7 with two dice (1+6, 2+5, 3+4, 4+3, 5+2 and 6+1).

Odds of 5 to 1 means that failure to roll a 7 happens five times as often as success. In the long run five of every six tries will fail and one will succeed. Therefore, odds of A to B against an outcome means that the probability of that outcome is:

$$\frac{B}{(A+B)}$$

If the odds against the favourite in a horse race are 3 to 2 this is equivalent to the horse having a probability of $\frac{2}{5}$ of winning.

Still throwing the dice, the odds on throwing a 4 are 1 to 5, while the probability of getting a 4 is 1/6. When you place bets on a horse, or a sport game, bookies will quote you, for example 10 to 1, meaning the odds on that horse winning is 1 to 10, while the probability of the horse winning is.

1/the number of horses racing

Formative assessment

Each learner gets an opportunity to throw a dice once. Before any one throws, do the following:

Predict how many times the dice will land on 4	
Predict how many times the dice will land on 6	

Note the number of times the dice falls on a number in the frequency table below, while the dice is being thrown.

Outcome	Frequency
1	
2	
3	

4	
5	
6	

Write the letters of the words VERY WELL on 8 cards and put them in a bag. Shuffle them well. Do the following:

What is the probability for a V to be drawn?	
What is the probability for an E to be drawn?	
What is the probability for an R to be drawn?	
What is the probability for a Y to be drawn?	
What is the probability for a W to be drawn?	
What is the probability for an L to be drawn?	

Divide into groups of eight. Each member draws a card. Note which letter it is. Put the card back and another member of the group draws a card. Note which letter it is. Continue until all the members of the group have drawn a card.

Mutually Exclusive Events

In simple terms, two events are mutually exclusive if they cannot occur at the same time (i.e. they have no outcomes in common). In short, mutual exclusivity implies that at ***most*** one of the events may occur.

In logic, two ***mutually exclusive*** propositions are propositions that logically cannot both be true. If an event X means that another event Y does not take place, X and Y are called mutually exclusive.

For example, when tossing a coin, it can only land on either head or tails, not both. The two events are mutually exclusive.

The probability of occurrence of two or more mutually exclusive events is obtained by adding the probabilities of the individual events. If the sum is equal to 1, we call them complementary events.

Formative assessment

Suppose you write the letters of the words VERY WELL on 8 cards and put it in a bag.

Shuffle them well and draw a card and note which letter it is.

What is the probability of drawing a card with a vowel on it from the bag?	
What is the probability of drawing a card with a consonant on it from the bag?	
What is the sum of the probabilities of drawing a vowel and drawing a consonant? (These events are mutually exclusive.)	

Independent Events

In probability theory, to say that two events are ***independent,*** intuitively means that the occurrence of one event makes it neither more nor less probable that the other occurs. For example:

- ✓ The event of getting a 6 the first time a die is rolled and the event of getting a 6 the second time are ***independent***.
- ✓ By contrast, the event of getting a 6 the first time a die is rolled and the event that the sum of the numbers seen on the first and second rolls is 8 are ***dependent***.
- ✓ If two cards are drawn *with* replacement from a deck of cards, the event of drawing a red card on the first trial and that of drawing a red card on the second trial are *independent*.
- ✓ By contrast, if two cards are drawn *without* replacement from a deck of cards, the event of drawing a red card on the first trial and that of drawing a red card on the second trial are *dependent*.

When ***two*** coins are tossed, the probability that one will land on heads is ½ and the probability that the other one will land on heads is also ½.

These two events are ***not mutually exclusive***. They are ***independent***. The probability of the occurrence together of two or more independent events is obtained by multiplying the probabilities of the individual events.

Formative assessment

Make a list of all the possible outcomes if two coins A and B are tossed.

If two coins are tossed 50 times, how many times do you expect to get two heads?	

If two coins are tossed 50 times, how many times do you expect to get one tail and one head?	

Make a list of all possible outcomes if two dices X and Y are rolled simultaneously.

Make a list of the different outcomes of which the total is 6.	
What is the probability that, if two dice are rolled simultaneously, the total is 6?	

UNIT STANDARD 7456

Unit Standard Title

Use mathematics to investigate and monitor the financial aspects of personal, business and national issues

NQF Level

3

Credits

5

Purpose

This unit standard will be useful to people who aim to achieve recognition at some level in Further Education and Training or to meet the Fundamental requirement of a wide range of qualifications registered on the National Qualifications Framework.

People credited with this unit standard are able to:

- ✓ Use mathematics to plan and control personal and/or household budgets and income and expenditure.
- ✓ Use simple and compound interest to make sense of and define a variety of situations including investments, stokvels, inflation, appreciation and depreciation.
- ✓ Investigate various aspects of financial transactions including costs, prices, revenue, cost price, selling price, loss and profit.

Learning Assumptions

Learners accessing this Unit Standard should be competent in Mathematical Literacy and Communications at NQF level 2.

Specific Outcomes and Assessment Criteria

Specific Outcome 1: Use mathematics to plan and control personal, regional and/or national budgets and income and expenditure: Bank accounts, provincial and key elements of national budgets and tax

Assessment Criteria

- ✓ Use mathematics to plan and control personal, regional and/or national budgets and income and expenditure
- ✓ Plans describe projected income and expenditure realistically
- ✓ Calculations are carried out using computational tools efficiently and correctly and solutions obtained are verified in terms of the context

- ✓ Budgets are presented in a manner that makes for easy monitoring and control
- ✓ Actual income and expenditure is recorded accurately and in relation to planned income and expenditure. Variances are identified and explained and methods are provided for control

Specific Outcome 2: Use simple and compound interest to make sense of and define a variety of situations***:*** Effective and nominal rates, commission, appreciation and depreciation

Assessment Criteria

- ✓ Use simple and compound interest to make sense of and define a variety of situations
- ✓ The differences between simple and compound interest are described in terms of their common applications and effects
- ✓ Methods of calculation are appropriate to the problem types.
- ✓ Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem
- ✓ Solutions to calculations are used effectively to define the changes over a period of time

Specific Outcome 3: Use mathematics to debate aspects of the national economy: tax, productivity and the equitable distribution of resources

Assessment Criteria

- ✓ Values are calculated correctly
- ✓ Mathematical tools and systems are used effectively to determine, compare and describe aspects of the national economy
- ✓ Debating points are based on well-reasoned arguments and are supported by mathematical information

Unit Standard Essential Embedded Knowledge

- ✓ Budgets
- ✓ Terminology and definitions associated with financial situations
- ✓ Estimation and approximation
- ✓ Compound increase and decrease

USE MATHEMATICS TO CONTROL BUDGETS

Outcome

Use mathematics to plan and control personal, regional and/or national budgets and income and expenditure

Assessment criteria

- ✓ Plans describe projected income and expenditure realistically
- ✓ Calculations are carried out using computational tools efficiently and correctly and solutions obtained are verified in terms of the context
- ✓ Budgets are presented in a manner that makes for easy monitoring and control
- ✓ Actual income and expenditure is recorded accurately and in relation to planned income and expenditure. Variances are identified and explained and methods are provided for control

Finance

Finance is basically interpreted as being able to manage money. The financial world along with all other industries have a language all of their own. We need to become aware of what all this jargon means to help us manage our money successfully. Governments have to formulate economic policies which aim to benefit the whole community. Therefore they often intervene in the commercial world to impose conditions which are necessary to achieve those objectives. These include measures to improve the balance of payments, to reduce unemployment, and to control the supply of money and credit.

Even though you may not be earning much, some of your income should be put aside and saved. It is all very well to convince yourself that starting your savings program can wait until you earn more, but by doing that you may never start. "Saving is a habit", a healthy one at that! For people who cannot start a savings habit, another option is to have money automatically deducted from your pay.

We all need some money stashed away for that rainy day, a safety net of three month's take home pay is advisable. This money should be placed in a term savings account with a bank, provided of course you can draw on this without waiting for months. Every month you should check your income against debts and expenditure. You'll be surprised at the incredible fluctuations that can occur. Also you are checking against running in the "red". There are three stages in your life when savings/ investing are at an optimum. They are when you cohabit a lodging with another person and share expenses, when you are living at home and are a wage earner, or when your children have finally left the nest, you are both working and your mortgage is greatly reduced.

Money governs what can be done in any situation. More careful management of money should always provide better value for the rand spent, but even more than that, an awareness of monies available, will enable the manager to make informed decisions on what can be afforded at any one time.

There are two aspects to financial management:

- ✓ The **BUDGET**: this is a plan of what can and should be spent over a given period of time, and what things should be allocated what amount of money.
- ✓ The **FINANCIAL RECORDS:** a system of books or computer files needs to be managed, keeping track of what is spent when and where; paying accounts, making purchases, authorizing and making payments, receiving money, sending bills to customers etc.

Budget

A budget is a plan of the amount of money coming in and the amount of money going out. In other words, it is a written plan of all the money you are earning and spending.

To work out the total amount of money that is coming in each month (your monthly income), write down all your earnings (your take-home pay), all your partner's earnings (your partner's take-home pay) and any other money coming in for the month.

Monthly Income	
Husband	
Wife	
Other	
Total Income	

It stands to reason that you should not spend more than you earn, if you do you will get into debt. There are three basic types of expenses: fixed expenses, changing expenses, and luxury expenses.

Fixed Expenses

These are expenses that have to be paid every month, and the amounts don't change, for example: rent.

Changing Expenses

These are expenses for things that you can't normally do without, and the amounts change from month to month, for example: petrol.

Luxury Expenses

These are items that you can cut down on. They are usually luxury items, for example: cigarettes.

Drawing Up A Budget

To begin drawing up your budget, list all the main expenses that you have to pay every month.

These include:

- ✓ Rent / Bond
- ✓ Insurance

- ✓ Electricity
- ✓ School Fees
- ✓ Hire Purchase Agreement

The next step is to list other money spent on items such as:

- ✓ Food
- ✓ Petrol
- ✓ Electricity
- ✓ Taxi or Bus Fare
- ✓ Telephone Calls
- ✓ Savings

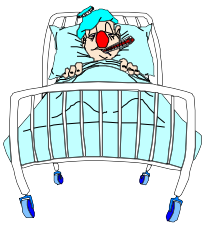
Then you list all the luxury items that you spend money on. These can include:

- ✓ Cigarettes
- ✓ Alcohol
- ✓ Entertainment
- ✓ Sports
- ✓ Clothes

Follow these steps when paying your monthly expenses:

- ✓ Pay your fixed expenses.
- ✓ Pay your changing expenses.
- ✓ Put some money into a savings account.
- ✓ Whatever is left over, use for luxury items

Reasons For Budgeting



The major reasons for budgeting for your expenses and savings are:

- ✓ to avoid getting into money trouble
- ✓ to prepare for a future where money won't be a problem
- ✓ to have "in-case" money for emergencies

Why should you save?

The money you save is used to pay for things that happen that you haven't planned for. It is wise to always have savings for repairs to the house, the car, or any other type of emergency.

If you have savings in the bank, you will not get into money trouble. This will mean you will not have to borrow money from the bank.

Drawing up your own budget

Formative assessment

- ✓ In the estimate column, write down your expenses.
- ✓ Write down your income.
- ✓ Total your expenses. Subtract this from your income

Expenditure	Estimate	Actual	Date paid
Fixed Expenses			
Bond/Rent			
Rates			
House Insurance			
Car Insurance			
Hire Purchases (e.g, fridge, furniture)			
School Fees			
Changing Expenses			
Groceries			
Petrol			
Electricity/Water			
Taxi Fare			
Telephone			
Retirement Annuities			
Savings			
Luxury Expenses			
Clothes			

Alcohol			
Cigarettes			
Entertainment			
Sports/Hobbies			
Donations			
SUB TOTAL			
Unforeseen Costs			
A. TOTAL SPENDING			

Monthly income	
Husband	
Wife/Partner	
Other	
B. Total income	
C. Subtract expenses from income	
Shortfall / amount over	

Define a budget in your own words.

Explain the following:

Fixed Expenses	
Changing Expenses	
Luxury Expenses	

National And Provincial Budgets

Just as you should have a budget, every business and state organisation has a budget. During the National budget of the Minister of Finance it is announced what the budget for education will be. This is the amount of money available to the Minister of Education to be spent on education. Of course, all the government departments are allocated funds in order to run their departments. Examples are: Transport, Defence, Social Services, SA Police Services, and so on.

The Minister of Education will then budget (plan) to indicate which portion of her budget will go to which section of education and how much money should be allocated to each province. A portion of the money available to her will be held back by the National Department of Education for their own expenses and special projects.

The various provincial departments of education must draw up budgets to submit to the National Department of Education, showing how they spent their money during the previous year and how much money they will need during this year to pay their expenses and to use for special projects, such as subsidised bus fares, new schools or upgrading of the facilities of existing schools. The money that is available will then be allocated to the various provinces, based on the requirements of their individual budgets.

As the financial year proceeds, the National Department of Education as well as the Provincial Departments of Education must manage and control expenses to ensure that they do not spend more money than was allocated to them. They must also ensure that the money is spent on the items as shown in their budgets. They cannot ask for money to build new schools and then spend the money on higher salaries for top management, for example.

Tax

The question remains, where does the Minister of Finance get the money to allocate to the various state departments?

The Government has to pay for many projects worth millions of rands, for the benefit of all South Africans. These projects include:

- ✓ Welfare
- ✓ Housing
- ✓ Education
- ✓ Health care, including hospitals
- ✓ Roads
- ✓ Railways

- ✓ Dams
- ✓ Policing

All the monies collected through Value Added Tax, SITE and PAYE and tax paid by companies, estates and trusts is paid over to the Receiver of Revenue. This money is then used to fund the various government departments.

VAT is paid every time you purchase goods such as groceries, cars, furniture, pay for water and lights, etc. A flat rate of 14% is added to the price of the goods and this amount of 14% is paid over to the Receiver of Revenue.

SITE and PAYE is the income tax you and other employees pay every month. This amount is deducted from your salary by the employer and paid to the Receiver of Revenue on a monthly basis.

S.I.T.E. stands for ***STANDARD INCOME TAX ON EMPLOYEES***. It is for people who earn less than R60 000 per year. If you do not have to fill in a tax form, your employer pays the amount of tax that you have to pay, directly to the Receiver of Revenue.

P.A.Y.E. stands for ***PAY AS YOU EARN***. If you earn more than R60 000 a year, you pay P.A.Y.E. You must fill in a tax return form. The tax year is normally from the first of March to the end of February each year.

All businesses pay a tax on their yearly profits, this is usually called company tax. This amount is calculated once a year, based on the net profit the business has made, and is paid to the Receiver of Revenue once a year.

Every year the Minister of Finance calculates the money that was paid from various sources to the Receiver of Revenue, and on the basis of this figure works out the budget for the next year. It stands to reason that he uses the figure of the previous year and then estimates (forecasts) what amount will be paid to the Receiver of Revenue during the current year. He then uses the planned (forecast) amount to calculate his budget and to allocate money to the various government departments. This is why civil servants are called servants: their purpose is to serve the public, since the public finances all the salaries paid to employees of government departments.

Formative assessment

Where does the Minister of Finance get the money to allocate to the various state departments?

What does the government do with the money the collect through tax?

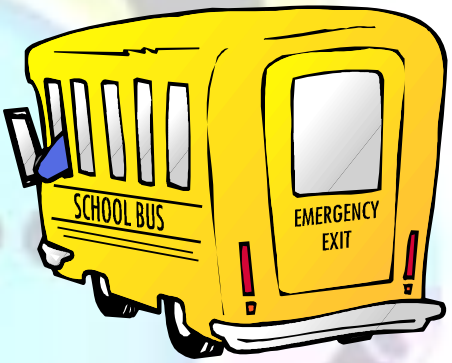
using, Education, Health care	

Control Of Income And Expenditure

Budgets

When we talk about National and Provincial budgets, budgeting usually consists of a financial plan that shows anticipated (what you think is going to happen) or planned (what you planned to happen) costs, revenues cash balances or net income.

Costs would be costs to run the various government departments and includes salaries, rental of office space, salaries, water and lights, telephone, the cost and maintenance of government cars, building of new premises, paying social grants such as old age pension, subsidising transport, etc. The list is very long and varies from department to department and province to province.



Revenues would be the monies collected from various forms of tax.

Cash balances and net income normally refer more to privately owned businesses than government departments and would be the money that is left over after all the expenses have been paid.

As with your personal budget, a government department on both provincial and national level uses a budget to plan and control expenses (expenditure). It is crucial that persons, businesses, governments (local and national) and all business ventures budget to plan and control expenditure according to projected income. You cannot spend more money than you receive – if you or the national or provincial government departments do this, you and they will run into debt. In the case of national or provincial government departments, it means that:

- ✓ pensions cannot be paid,
- ✓ school bus operators cannot be paid,
- ✓ the books required for schools cannot be bought and delivered to individual schools and so on and so on.

A major benefit of budgeting is that it requires managers to scrutinise and coordinate their operations very carefully, to ensure that they do not overspend. If done properly, this will help management to anticipate, many crises.

Another benefit of budgeting is that managers can and should compare actual costs (expenses) with the costs allocated in the budget – in the same way that you compare your

expenses with your budget every month. If there are big differences between the budgeted costs and the actual costs, management can do what is necessary to stop this from happening.

This is why budgets are looked at each month, just like you should look at your budget every month to make sure that you do not overspend.

Managers mainly use two types of documents to control the budget: a Budget Management Report and an Income Statement.

	Month	Year	Income				Purchases				Expenses			
			Budget	Actual	Over	Under	Budget	Actual	Over	Under	Budget	Actual	Over	Under
1	March	2003	4 000	3 600	-	400	2 000	2 100	100	-	1 500	1 800	300	-
2	April	2003	5 000	4 200	-	800	2 500	2 100	-	400	1 000	1 100	100	-
3	May	2003	6 000	5 100	-	900	3 000	3 000	-	-	1 000	1 100	100	-
4	June	2003	6 000	6 300	300	-	3 000	3 000	-	-	1 000	1 100	100	-
5	July	2003	6 000	7 000	1 000	-	3 000	3 000	-	-	1 000	1 200	200	-
6	August	2003	6 000	7 000	1 000	-	3 000	3 400	400	-	1 000	1 200	200	-
7	Sept	2003	7 000	7 500	500	-	3 500	4 000	500	-	1 000	1 200	200	-
8	October	2003	7 000	6 500	-	500	3 500	4 000	500	-	1 000	1 300	300	-
9	Nov	2003	7 000	8 000	1 000	-	3 500	4 000	500	-	1 200	1 500	300	-
10	Dec	2003	8 000	8 000	-	-	4 000	4 100	100	-	1 300	1 600	300	-
11	Jan	2004	6 000	4 000	-	2 000	3 000	3 000	-	-	1 000	1 200	200	-
12	Feb	2004	7 000	6 000	-	1 000	3 500	3 000	-	500	1 000	1 200	200	-
Total			75000	73200	-	1 800	37500	38700	1 200	-	13000	15500	2 500	-

The previous page contains an example of a Budget Management Report. From this report, we can see that:

- ✓ The income was R1800 less for the year than anticipated
- ✓ The purchases made was over budget by R1200 for the year
- ✓ The expenses were R1300 over the budgeted amount for the year
- ✓ The total income was R73200 for the year
- ✓ The total amount spent as purchases and expenses was R54 200
- ✓ There was still a balance (profit) of R19000 left after everything was paid.

In the case of a business, this is a good thing, since a business exists in order to make a profit. In the case of a government department it might not be such a good thing. The surplus money could have been used to pay for special projects such as housing, hospitals, schools, roads, etc.

Following is an example of a budget report that gives a breakdown of monthly expenses. The management budget report is compiled from various such smaller reports that give breakdowns of expenses per department, purchases per department, etc.

March 2003					
	Payments and Expenses	Budget	Actual	Over	Under
1	Vehicle Licence	R54	R54	-	-
2	Vehicle Loan	R300	R300	-	-
3	Petrol	R200	R250	R50	-
4	Owner's Salary	R600	R600	-	-
5	Wages	R346	R596	R250	-
6					
7					
8					
9					
10					

11					
Totals		1 500	1 800	300	-

The income statement can also be used to see the variance (difference) between actual expenditure and budget.

An Income and Expenditure Statement is a record of all income and expenditure with which the business conducted business in the year. This statement will tell you whether you have made a profit or a loss over a period, normally twelve months. As the name suggests, it shows you the income received and the expenses paid to operate the business.

Below is an example of an income statement.

INCOME STATEMENT – Actual vs. Budget: 01/04/2006 to 31/03/2007

	Actual	Budget	Variance	
			Value	Percentage
SALES	100 000.00	80 000.00	20 000.00	0.20
COST OF SALES (Purchases)	75 000.00	60 000.00	15 000.00	0.20
OTHER INCOME	100.00	66.00	34.00	0.34
EXPENSES	14 300.00	16 400.00	-2 100.00	-0.15
Bank Charges	1 000.00	800.00	200.00	0.20
Cleaning	500.00	500.00	-	-
Consumables	200.00	300.00	-100.00	-0.50
Entertainment Expenses	1 500.00	1 600.00	-100.00	-0.07
Interest Paid	100.00	200.00	-100.00	-1.00
Printing & Stationary	3 000.00	3 000.00	-	
Telephone & Fax	5 000.00	6 000.00	-1 000.00	-0.20
Vehicle Expenses	3 000.00	4 000.00	-1 000.00	-0.33

NET PROFIT/(LOSS) BEFORE TAX	10 800.00	3 666.00	7 134.00	0.66
TAX	3 780.00	1 283.10	2 496.90	0.66
NET PROFIT/(LOSS)	7 020.00	2 382.90	4 637.10	0.66

From this we can see the following:

- ✓ How much money came into the business
- ✓ How much money was spent to purchase raw materials
- ✓ How much was spent on various expense items
- ✓ What the net profit before tax was
- ✓ What the net profit after tax was
- ✓ What the difference was between what was budgeted for and what actually happened for all the above items

Can you see how managers use these reports to ensure that the expenses of the business or the government departments do not become more than the income that was received?

You could do a similar exercise with your household budget, where profit would be money that can be saved.

Formative assessment

List two benefits of budgeting

Which financial records does a manager use to control income and expenses?

Refer back to the Management Budget Report for the year 2003 and answer the following questions:

What was the total income for the year?	
What was the difference between the budgeted income and the actual income?	
How much was spent on purchases for the year?	
What was the difference between the budgeted expenses and the actual expenses?	

Refer back to the payments and expenses report for March 2003 and list the expenses that are indicated on the report.

List at least four other expenses you can think of that a business or government department will have.

Profit and Loss

A business exists with the purpose of making a profit. To be able to make profit, there will be expenses (expenditure). If the income (generated from sales or services) is more than the expenses, a profit was generated. A profit can only be generated if the expenses are managed carefully with the use of a budget.

Profit, in other words, it is the amount left over after a business has paid all its bills.

When a business makes a loss, it will go out of business.

There are many things that determine profit or loss in a business:

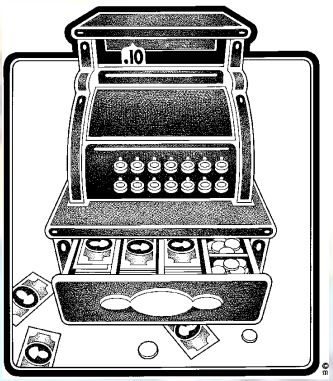
- ✓ The price at which goods are sold
- ✓ The price at which raw materials are purchased

- ✓ How many goods are sold
- ✓ Control of expenses

Are some of the factors that influence profit.

Price

The price at which a business sells its goods or services, the selling price, should always be more than the total cost price. If not, the business will break even- there is no reward/returns to the business owners for investing their money in the business; or the business will make a loss – the business owners have to invest more money into the business to keep the business going.



The difference between cost price and selling price is called the mark up. Mark up is always calculated on the cost price. The mark up should be high enough to cover the cost price and overheads, plus allow for a profit.

Cost price
+ Expenses
+ Profit
= Selling Price

Profit is always expressed as a percentage of the selling price, because you only get the profit once the product has been sold.

Cost: R 1,00

Mark-up: R ,50

Selling Price: R 1,50

Mark-Up Percentage And Profit Percentage

Mark up is always greater (in percentage terms) than margin (profit), although the monetary figure is the same.

$$\text{Mark Up} = \frac{\text{Selling Price - Cost Price}}{\text{Cost Price}}$$

Mark up % = (,50 divided by R 1,00) = 50%.

$$\text{Margin} = \frac{\text{Selling Price - Cost Price}}{\text{Selling Price}}$$

Margin (Profit) % = (,50 divided by R 1,50) = 33,3%.

Formative assessment

How should a selling price be worked out?

If your total cost of a case of wine (with twelve bottles) is R 120. What is the cost of one bottle of wine?

You have decided to add a mark-up of 100% to the wine. At what price will you sell the wine?

As can be seen from the previous section, the income statement shows the profit or loss made by the business.

What does a business do with profits?

A portion of profit is used to pay the shareholders in the company, as they also have to get a reward for the money they invested in the business.

Another portion is used for investments and working capital. The biggest portion of the profit is reinvested in the business so that the business can, for example, buy new machinery, expand the range of products or services and other similar activities that are aimed at making the business bigger and more profitable.

Some companies use a portion of the profit to reward their employees by paying out bonuses

SIMPLE AND COMPOUND INTEREST

Outcome

Use simple and compound interest to make sense of and define a variety of situations

Assessment criteria

- ✓ The differences between simple and compound interest are described in terms of their common applications and effects
- ✓ Methods of calculation are appropriate to the problem types
- ✓ Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem
- ✓ Solutions to calculations are used effectively to define the changes over a period of time

Interest

Simple and Compound Interest

Interest is the growth of your investment or asset, an example is that you deposit R100.00 at a bank for a period and receive in return interest on the amount deposited.

Interest can be seen in two ways:

- ✓ It is money paid for the use of money lent e.g. a loan at a bank, a bond on a house, a loan for a car etc.
- ✓ It is money earned for placing it in an investment, savings account, debenture etc.

Interest rates differ from bank to bank, and usually the interest you pay for a loan is higher than the interest you would earn from an investment or savings account

Interest is the cost of money; it is money which is paid for the use of money.

Basic terminology

Simple Interest	Interest earned/paid for a short period of time e.g. a month, 2 months, usually charged on overdue accounts.
Compound Interest	Interest earned/paid for long periods of time i.e. 12 months or more, usually for home loans, cars, investments, debentures, etc.
Appreciation	Increase in monetary value e.g. property investments.
Depreciation	Decrease in monetary value e.g. a car's value is reducing every year.
Commission	Money paid to an agent in a commercial transaction e.g. a sales person earns commission on sales made.
Nominal Interest	The annual rate at which many loans and financial instruments are quoted is the nominal interest rate.
Effective Interest	The effective interest rate adjusts the nominal interest rate based on the amount of compound interest charged/earned and the number of days assumed in a year.

Simple interest

The calculation of interest may be '***simple***' or 'compound'. ***Simple*** interest is the application of a percentage rate to the principal sum for the period in question, as expressed in equation

$$i = \frac{p \times t \times r}{100}$$

Where ***i*** is the actual interest, ***p*** is the principal sum, ***t*** is the time or period of the loan in years and ***r*** is the percentage rate of interest.

Suppose that a person walks into a bank and deposits R1 000 in a savings account that earns an interest rate of ten per cent per year. How much will this depositor have in two years time if the account earns simple interest? The amount of interest is calculated by assuming that the interest does not earn interest. In other words the same base amount or principal must be used for each year. For this deposit, the value of the account after two years will be equal to the sum of the principal and the interest earned over the two years.

The interest earned in the first year is R 1000 x 0, 1 = R 100.

In the second year a similar amount of interest is earned, therefore at the end of two years the value of the account is RI 000 + R100 + RI00 = RI 200

Lindiwe has opened a Sales House clothing account. She has spent R1 080.00 on her account. Her minimum monthly payments are R200.00. Unfortunately for the past 2 months Lindiwe has been unable to pay this account, as she has had other expenses that she has had to cover. When she received her statement, she realised that Edgars have charged her 7% interest on her overdue account per month.

This is an example what her statement will look like:

SALES HOUSE (PTY) LTD		STATEMENT
P O Box 12406		Date: 30/11/04
Johannesburg, 2000		Page: 1
Tel: 011 123 4567		Account Number:AC5280
Fax: 011 123 4589		
MISS LINDIWE MAHLANGU		
P O Box 20178		
Kempton Park		
2263		

Account	Date	Page
AC5280	30/11/05	1

Date	Reference	Description	Debit	Credit	Amount
30/09/04		BROUGHT FORWARD	1,080.00		1,080.00D
01/10/04	PN0904	Minimum payment due	200.00		
30/10/04	INT0904	Interest Charged	75.60		1,155.60D
01/11/04	PN1004	Minimum payment due	200.00		1,155.60D
30/11/04	INT1004	Interest Charged	80.89		1,236.49D
01/12/04	PN1104	Minimum payment due	200.00		

120+ Days	90 Days	60 Days	30 Days	Current	Amount Due:	1,236.49
0.00	0.00	280.89	275.60	200.00	Amount Paid:	

Payments taken into account up to 05-12-04	Total Due	Comments: We noticed that your account is in arrears and need to draw your attention to regular monthly installments. This may affect your credit rating with the Credit Bureau.
	1,236.49	

Deductions made from this statement:

- Lindiwe has been charged 7% interest for every month that she has not paid. The interest has been charged on the outstanding amount. Because Lindiwe missed the second month, interest was charged again, on the new amount, not on the balance brought forward, so due to her inability to pay, she is now paying more than she should.
- If Lindiwe had paid her account, she would be closer to having credit in her account so that she could purchase more goods, but now she does not, and only owes more money.
- After 120days (4 months) of not paying, Lindiwe is going to be handed over to their legal department, and more fees will be incurred. This has a serious impact on her credit record and may result that she will not be able to obtain other accounts or be able to purchase on credit.

What can be seen from this statement?

- The total balance owing by Lindiwe is R1 236.49
- Due to not paying, she has been charged simple interest to the value of R156.49

Amount due on 01/12/04	R1 235.49
Less opening balance	R1 080.00
Simple interest charged	R 156.49

- If Lindiwe had paid her minimum payment due for the past 2 months, her closing balance would have been R680.00

Amount due on 30/09/04	R1 080.00
Less payment on 01/10/04	R 200.00
	R 880.00
Less payment on 01/11/04	R 200.00
Balance due on 01/12/04	R 680.00

Compound interest

Compound interest means interest paid on interest. If you leave your interest in your account, your money will grow at a much quicker rate. Our money becomes more valuable over time.

In most cases, in the financial world, interest is calculated as compound interest. With this method both the principal amount and the interest earn interest in later periods. In other words, the amount principal plus interest for one period, becomes the principal for the next period

Compound interest is interest on the principal sum, plus the accruing interest, as expressed in the following equation for interest compounded annually:

$$i = p(1+\frac{r}{100})^t - p$$

Interest on cheque and savings accounts at banks is usually 'simple' interest but is compounded in the case of investment accounts where interest is paid on interest previously credited to the account. The 'rests' between interest dates are critical, since the compounding effect of interest paid yearly in arrears is not as good as the compounding on interest paid quarterly.

In the case of an overdrawn current account, interest is calculated from the actual date when a customer's cheque is paid (not from the date on the cheque), and from the actual date when money is credited to the account (usually at the computer centre).

Compound interest is earned/charged on money invested/loaned over long periods of time i.e. more than a year.

Compound interest increases the value of your money over time. If you were to invest money over 5 years, the interest earned on the money you have invested would have increased. So the value of your money after 5 years is more than the value of your money today.

When you invest money at compound interest, an amount doubles itself:

At 7 percent in	10 years	89 days
6	11	327
5	14	75
4.5	15	273
4	17	246

At simple interest, an amount doubles itself:

At 7 percent in	14 years	104 days
6	14	239
5	20	-

4.5	22	81
4	25	-

EXAMPLE: Compound interest

A man invests R5 000 in a company at 13% interest per year for 3 years. He decides not to use any of the interest he earns until the end of the investment period. How much will he have earned after 3 years?

Solution:

Amount at the beginning of the 1st year	R5 000
Interest(13% of R5 000)	R650
Amount at the beginning of the 2nd year	R5650
Interest (13% of R5 650)	R734.50
Amount at the beginning of the 3rd year	R6 384.50
Interest (13% of R6 384.50	R829.99
Amount at the end of the 3rd year	R7 214.49

EXAMPLE: Calculating compound interest using the formula

calculate the compound interest on an investment of R678 which is invested for a period of 12 years at 15,6% interest per year.

Solution

$$i = p(1+\frac{r}{100})^t - p$$

$$i = 678(1+\frac{15.6}{100})^{12}$$

$$i = R3891.21$$

Interest = Final amount – Initial amount

Interest = R3 891.21 - R678 = R3 183,21

The following deductions can be made about compound interest:

- ✓ The compound interest earned annually increased the initial value of the money.
- ✓ When interest is compounded annually, it is compounded on the value of the money at the end of every year, not on the value of the money at the start.

Formative assessment

Which of the following two investments are the most lucrative? R500 invested for 10 years at 14% simple interest per year
R500 invested for 10 years at 13% compound interest per year

Effective and Nominal Interest Rates

The annual rate at which many loans are quoted is the ***nominal interest rate***. One may, for an example, make an investment that pays interest at a nominal interest rate of 10% annually.

The ***effective interest rate*** adjusts the nominal interest rate based on the number of times the interest is compounded and the number of days assumed to be in a year.

Financial managers and investors must be careful when comparing different interest rates. They have to seek the cheapest financing available. Therefore the effective rate should be used when evaluating costs and returns. Often the interest rates charged by banks and other financial institutions on savings accounts are effective interest rates. Home and commercial loans state the nominal rate. If the interest charged is only compounded annually (like in the previous example), then the effective rate is equal to the nominal rate. The more regularly the interest is compounded in a year, the higher the effective interest rate will be.

Before you take out a loan from a bank for a home or a car, it is vitally important that you determine what interest rates they will be charging you, and how much additional money you will be paying for your home/car.

Example 1

A financial manager is evaluating two loans and the effective interest rate charged. Second National Bank (SNB) quotes a nominal interest rate of 16,5% compounded semi-annually. Third National Bank (TNB) quotes a nominal rate of 16,4% compounded daily.

Which of the two loans should the financial manager select in order to obtain the lowest effective interest rate cost?

SNB:

Effective interest rate $= \left(1 + \frac{0{,}165}{2}\right)^{2} - 1 = 0{,}17181 = 17{,}18\%$

TNB:

Effective interest rate $= \left(1 + \frac{0{,}164}{365}\right)^{365} - 1 = 0{,}17817 = 17{,}81\%$

Based on the objective of obtaining the lowest effective cost of financing, the loan offered by Second National Bank should be used.

The following deductions can be made from this example:

- ✓ Originally one might have thought that TNB are offering a better interest rate (16,4%), but after working it out, SNB's 16,5% is a better rate in the long run.
- ✓ It is important that we ensure that even though the rate offered by the bank may seem lower, it actually is not.

The ***real rate of interest*** is the percentage increase in purchasing power that the borrower pays to the lender for the privilege of borrowing. It indicates the increased ability to purchase goods and services that the lender earns.

The ***nominal rate of interest*** is the percentage by which the money the borrower pays back exceeds the money he borrowed, making no adjustment for any fall in the purchasing power of this money that results from inflation.

Depreciation And Appreciation

Depreciation

Depreciation is the decrease in monetary value of your assets e.g. a car.

Depreciation is a reduction in accounting earnings which are intended to reflect the reduction in value of an asset. In other words, the car, furniture, clothes, cell phones and computer equipment you buy decrease in value every year, meaning they are worth less than you paid for them. Depreciation occurs when your assets, including the buying power of your money, loses value.

Example

Solly bought a new car 5 years ago, it cost him R35 000. He has decided that he would like to buy a bigger car so that he can do deliveries. Solly took the car to the dealership, and they offered him R15 000 for his car.

Solly is angry! He knows he has lost R20 000 because he bought the car for R35 000 and now can only sell it for R15 000!

Deductions made from this example:

- ✓ A car is not an asset, it depreciates in value.
- ✓ A few causes of depreciation are:
  - ✓ High mileage
  - ✓ Poor maintenance
  - ✓ Accidents that have been repaired
  - ✓ Newer, better models of the same cars coming out

- ✓ High demand of cars
- ✓ Solly in effect has lost R20 000 and all the interest that he paid to the bank for the car.
- ✓ Solly's car depreciated with R4 000 per annum: R20 000 / 5 = R4 000

Appreciation

Appreciation, on the other hand is when assets, including the buying power of your money, increases in value. Property such as houses and flats are seen as assets that increase in value.

Interest is sometimes seen as a method of protecting your money against losing buying power due to inflation, provided you let the interest accumulate and don't use it.

Example

Mpho bought a home 5 years ago, she paid R56 000 for her house. As her family has grown and she requires two more rooms, Mpho has decided to buy a new house.

Mpho contacted her bank and has requested that a property evaluator comes round to evaluate her house, before she tries to sell it. The evaluator let Mpho know that her house is now valued at R78 000.

Deductions made from this example

- ✓ Mpho's house has appreciated in value by R22 000:

  R78 000 – R56 000 = R22 000
- ✓ Mpho's house has increased in monetary value by R4 400 per annum

  R22 000 / 5 = R4 400
- ✓ Mpho has made R22 000 after 5 years
- ✓ Mpho can sell her house and use the R22 000 to put down a deposit on a bigger house.
- ✓ Property usually appreciates in value.

Inflation

In its simplest form, inflation is an unhealthy and steady increase in the price of goods and services. The end result of inflation is a steady erosion in the buying power of your money. It affects all of us; it will make all of us poorer over time. It will eventually reduce our standard of living, it will destroy our confidence and peace of mind. Speak to anyone who retired on a fixed income in the last ten to fifteen years. At an inflation rate of 15%, the purchasing power of your money halves every 4,8 years.

Magnus Heystek's story of the disappearing R50 note:

He explains that R50.00 now will still be R50.00 in five years' time, but we will be able to buy much less with the same R50.00.

R50 in itself is not worth anything. It's what you can do with it in the future that is important.

So, the biggest threat to your money is something called INFLATION.

The purchasing power of money is one of the most important concepts when it comes to formulating an investment plan for yourself.

All investments must be geared towards protecting the purchasing power of your money.

The effect of inflation is that the value of the rand is dropping. Thus the value of assets rises as the purchasing power of the rands in which they are valued drops. But inflation is not bad for everybody - for every person who loses by inflation, another will gain.

The value of the average family home rises by 12% per annum due to inflation. The winners are present home-owners who see the value of their asset increase. The losers are those who have not yet bought, as it now costs them more to buy. Of course, the last couple of years has seen an increase of more than 20% per year in the value of houses and flats.

To beat inflation, one has to earn a real rate of return which means a return that beats the inflation rate after taxes. The successful investor is one who always strives to earn a rate of return equal or above the inflation rate.

Investments in the so-called "traditional" category of investments in banks have, on average, lost 5% of the purchasing power of their money per annum, over the last 20 years or more. By investing in equities and mortgage bonds, your chances of beating inflation are better.

So it is clear that in times of inflation it is sound strategy to keep the major part of your assets in a form which should increase in value, and only a small part in banks, building societies and other areas where it is losing value every day.

The Rule of 72

The Rule of 72 is a simple way to calculate the effects of inflation. If we take the number 72 and divide it by the expected inflation rate, the answer will be the number of years for money in cash form to lose HALF of its purchasing power or assets to DOUBLE in value.

If inflation was at 16% on average over the last 20 years (i.e. 72 ÷ 16 = 4,5), the following would have happened:

every four and a half years an asset such as an income-producing property (a property that you bought and rented out) would have doubled in price,

and the money you left as interest-bearing deposits would buy half the products it would have bought four and a half years ago.

If inflation goes to 20%, this doubling or halving will occur every three and a half years, (i.e. 72 ÷ 20 = 3,6).

If inflation goes to 6%, this doubling or halving will occur every 8 years

Let's see how inflation can affect a typical married retiree who leaves work with what appears to be a large sum of money. Bill retires at 60 years of age with R200 000 which he invests in fixed deposits at 15%. The return is R30 000 per year..

Let's look at what happens if inflation runs at 16%.

After four and a half years the purchasing power of the capital is down to R100 000 and the purchasing power of the annual income is down to R12 000.

In another four and a half years the capital is only worth around R50 000 and the income will buy only R6 000 worth of goods and services.

Certainly the face value of the fixed deposit would still be a nominal R200 000 but money is only worth what it can buy and its purchasing power has been slashed by 75% in just nine years.

Contrast this to the position of Jim who retired at the same age and used the R200 000 to buy income-producing property. The return of 10% after all costs meant that initially Jim's income was only R20 000 per year before tax, but due to inflation, grew every year. After nine years the property was worth nearly R800 000 and the income had risen to almost R80 000. By using inflation-beating techniques, Jim had managed to maintain his standard of living. On the other hand, Bill was forced to live on a decreasing (in real terms) income from his investment in fixed deposits.

Of course, currently inflation runs at rates much lower than 16%, meaning that the buying power of your money halves every 8 to 10 years, while your assets double every 8 to 10 years, but the basic lesson in the above case study is still the same:

In Bill's case: After 8 to 10 years the purchasing power of the capital is down to R100 000 and the purchasing power of the annual income is down to R12 000.

In Jim's case, after nine years the property will be worth nearly R400 000 and the income will have risen to R40 000 per year, giving him purchasing power of R20 000 per year. Jim will have maintained his standard of living, while Bill's income has steadily decreased.

Formative assessment

Use the rule of 72 and calculate in how many years the buying power of your money will halve if the inflation rate is 6%. Calculate in how many years the value of income earning assets will double if the rate of inflation is 12%

e of income earning assets will double in 6 years

THE NATIONAL ECONOMY

Outcome

Use mathematics to debate aspects of the national economy: tax, productivity and the equitable distribution of resources

Assessment criteria

- ✓ Use mathematics to debate aspects of the national economy: Range: Tax, productivity and the equitable distribution of resources
- ✓ Values are calculated correctly
- ✓ Mathematical tools and systems are used effectively to determine, compare and describe aspects of the national economy
- ✓ Debating points are based on well-reasoned arguments and are supported by mathematical information

Introduction to Economics

Economics studies how man obtains and uses scarce resources to produce goods and services, and how he/she distributes these goods and services to people to satisfy their unlimited needs (wants). Economics is a social science because it studies a part of man's behaviour.

When we deal with economics, we also have to deal with economic terminology. The table below lists and explains certain terms used.

Basic economic problem	The basic economic problem is that of scarcity as a result of unlimited wants. No country in the world has unlimited resources. As a result choices have to be made. If there were an unlimited supply of everything, we would not have to study economics. A very good example of limited resources in South Africa is water. The water that is available has to be shared between farmers who produce our food, industry who produce the things we want and need, such as iron and steel to build cars, and then water that ordinary people use daily to drink and clean.
Scarcity	Economics studies the problem of scarcity. There is not enough in the world to satisfy everyone's needs. The individual does not have enough means to satisfy all his/her needs.
Factors of production Collectively the resources listed below are called ***factors of production*** because they are used in the process of production.	
Land	Land not only refers to agricultural land, it also includes all natural resources such as minerals, oil deposits, coal deposits, climate, etc. Land is immobile and its supply is limited.
Labour	All the physical and mental efforts provided by people are called labour. Wages are the remuneration paid to an employee for services rendered
Capital	Capital is the contribution made by the owner by putting his/her capital at the disposal of someone else who requires it to further production, and can include buildings, equipment, vehicles, etc. The remuneration for the investor is called interest
Entrepreneur	The entrepreneur is the person who organises land, labour and capital to manage the day-to-day operations and bears the risk of the business. The compensation for his/her efforts is called profit. Profit is the excess of income over expenditure in an

	enterprise

Kinds of production

Commodities	The things that are produced by the factors of production are called commodities. Commodities are divided into goods and services: goods are tangible, as are cars and shoes; services are intangible, as are haircuts and education. Goods are valued because of the services they yield. A car, for example, is valued because of the transportation it provides, and possibly also for the flow of satisfaction the owner gets from displaying it as a status symbol. The total output of all commodities in one country over a period of time, usually taken as a year, is called its national product.
Production and consumption	The act of making goods and services is called production, and the act of using these goods and services to satisfy wants is called consumption. Anyone who helps to make goods and services is called a producer, and anyone who consumes them to satisfy his/her wants is called a consumer.

Government's role

Government's role is to manage the country and to create the best environment for the economy to grow. They do this by passing laws. Managing the country is not an easy task. South Africa is part of the world community and often what happens in one part of the world affects us.

Can you give an example of a world event which affected our economy and the value of our Rand?

Our Government has to respond to these forces and has to actively promote trade and encourage investment. We have to remember that we cannot force people to trade with us. There are other countries competing with us for markets, so we have to make sure that our goods are as competitive as possible.

What are some of the things we can do to be competitive?

Nor can we force countries or companies to invest in South Africa. Investors want a return on their investment.

How can we attract investors to South Africa so that our economy can grow

If our Government makes wrong decisions our economy will shrink and ultimately die.

Can you give examples of what is happening to the economies of some of our neighbours in Southern Africa?

Equitable Distribution Of Resources

The equitable distribution of resources is to make sure that everyone who needs resources, such as water, has access to these resources. This means that government must make sure that

- ✓ Farmers have water in order to provide us with the food we need
- ✓ Industry and businesses have water to produce goods that we can use and export
- ✓ General public have water to use on a day to day basis to drink and clean with

The country's new system of government is based on the Constitution of the Republic of South Africa Act, Act No. 108 of 1996. This Act also governs the entire public administration section, from parliament to local government.

Government Institutions are enterprises which are controlled exclusively by the state, and for which budgets are submitted to parliament. Government Institutions are represented by government departments such as Education and Justice.

Government Institutions have the Following Characteristics:

- ✓ They are controlled exclusively by the state.
- ✓ They are financed by means of budgets which are tabled in parliament. Once these budgets have been approved, the funds are made available to the departments concerned, via the Treasury.
- ✓ Expenditure must be accounted for to the state, in the person of the Auditor-General.
- ✓ Economy and productivity serve as measures of efficiency. This is because there is no endeavour to break even.
- ✓ The break even point is reached when costs are equal to revenue.

One of the most important developments in modern public administration has been the development of the budgeting system of government. The use of the financial resources of a society by government has become very important.

All government programs must be planned in advance and financial and other resources must be allocated to various activities in the system. Public administrators play a very significant role in the allocation and development of programs and as such they control important decision making roles. All public monies have to be accounted for and this has placed a vast responsibility on the public administrative sector.

The budgeting system, as it is known in modern times, began in Britain during the 19th century. The system works as follows:

The executive arm of government recommends how much should be allocated to various departments

The legislative arm of the government is responsible for the suitable distribution of those amounts to the various departments, and

The public administrative executive supervises those expenditures.

National Economic Planning

Any national economy in the modern world requires highly trained specialists to ensure that scarce resources are distributed and utilised equitably – where they are needed most, without ignoring the needs of other departments and provinces.

As mentioned previously, most of government's monies come from taxes that are paid by individuals, companies, estates and trusts. The various provincial and local governments have to compile budgets every year, giving details of how much money they need and what they will spend it on.

Some of the areas that are the responsibility of national, provincial and local governments will be discussed.

Energy And Resource Development

Government must provide sufficient energy and other resources that are required for the efficient running of the country, use by private businesses and industries, as well as use by individuals living in the country.

Government must plan for present and future use of these resources by stockpiling raw materials such as crude oil and other strategic materials relevant to the needs of a modern society, as well as ensuring that new plants are built to accommodate future needs.

Then government must ensure that the energy resources are distributed equally amongst the users so that everyone has enough to meet their needs.

Water Supply, Waste Disposal and Pollution

Similarly, government has the responsibility to provide the following services:

- ✓ Water
- ✓ Waste disposal
- ✓ Control of pollution

Once again, the present and future needs of the country must be planned for and government must ensure that these resources and services are distributed equally amongst the users: industries, businesses, private users.

Agricultural Development

In any society or nation, the planning and implementation of agricultural policy is of vital importance to the system. The agricultural system any nation enjoys priority, particularly since the feeding of a nation is dependent upon its successful production process.

This sector of the economies of developing countries is by far one of the most important for a variety of reasons. If one looks at the food production levels of many developing countries, one finds them to be of such a low level that emergency food imports have to be undertaken in order to avoid mass starvation. To relieve this problem, much more attention must be paid to 'he informal agricultural sector in these countries and the level of agricultural production must be raised. If surpluses are achieved the national economy can also benefit through the export of foodstuff, which in turn may bring in much needed foreign currency.

One way of doing this is through land reform, which will give all sectors of the public an opportunity to own land. Then, of course, smaller farmers should be supported by government in terms of funds to get started, support during the production process by means of training and government should also ensure that the small farmer can sell his surplus products on the general market.

Industrial Development

In order for industrialization to take place, infrastructural facilities should be provided. These include transportation facilities, electricity and other energy requirements, markets and other much needed facilities such as raw products.

Once again, all these resources should be distributed equally amongst the users, so that one party does not benefit at the cost of another party.

Welfare Benefits

Government must attempt to create a public assistance scheme that meets the needs of the society concerned. These needs must be based upon the resources available to the state to meet the welfare requirements.

In many societies the state takes full responsibility for the welfare needs of the community, while in others, private organizations also take part in relieving the more urgent needs of individuals. The state can also assist private organisations on a subsidy basis so that they may give service to certain groups such as unmarried mothers, the handicapped and so on. Often the state helps private institutions dedicated to educating handicapped or otherwise underprivileged members of society.

Health services are also of prime importance and as such, government departments are faced with the serious task of providing medical care for the public. In order to do this, government must be involved in the training process of medical personnel such as doctors, nurses and other paramedical personnel to man hospitals, clinics and other facilities.

Military and Police Facilities

These services render important services to societies. The purpose of the military arm of the security services is to serve and protect the public in the event of external attacks on a nation – attacks from outside the borders of the country.

The purpose of the police is to keep the public safe from criminal acts and also to keep the peace generally in any society in the event of civil unrest.

Financial Affairs Relating To The Provinces

Section 214 of the Act states that the provinces are entitled to an equitable share of the national revenue. This includes a portion of the income tax on individuals collected in the province and a percentage of the value added or sales tax collected within the province. The allocations made for the provinces are subject to the advice of the Financial and Fiscal Commission.

A province may, in terms of Section 228 of the Act, levy taxes, surcharges or levies, but there is a provision that the approval of Parliament and the recommendations of the Financial and Fiscal Commission is essential..

No province may levy taxes that are detrimental to the national economic good or national policies. Taxes levied by the provinces may also not negatively effect inter-provincial commerce or have a negative effect on the mobility of goods, capital, services or labour.

The Functions And Powers Of Local Government

In terms of Section 152(1) of the Act, a local government must:

- ✓ Provide democratic and accountable government for local communities
- ✓ Ensure the provision of services to communities in a sustainable manner
- ✓ Promote social and economic development
- ✓ Promote a safe and healthy environment
- ✓ Encourage the involvement of communities and community organisations in the matters of local government.

This means that the local government must provide

- ✓ Water
- ✓ Sanitation
- ✓ Transportation facilities
- ✓ Electricity
- ✓ Primary Health Services

- ✓ Education
- ✓ Housing and Security
- ✓ Security and Safety
- ✓ In an equitable manner.

Local government is entitled to charge for the services they provide – how would they get the money to provide the services otherwise

All governments in South Africa, local, provincial and national, must be transparent in everything they do. This is why their budgets are published every year and made available to the general public. In future, when you receive a copy of a government budget, read through it to find out what they are doing with your money.

Productivity

Productivity is the relationship between input and output.

Labour productivity is typically measured as output per worker or output per labour-hour.

What this means is that, if you can produce the number of items or even more per day or week than your fellow workers or than the manager budgeted for, you are productive. It costs management the same amount to produce the items, but they are getting more items, meaning they can make a profit.

Productivity is what makes the modern world go around. A company with employees that are not productive, will not survive and everyone will lose their jobs.

Companies can increase productivity in a variety of ways. The most obvious methods involve automation and computerisation which minimise the tasks that must be performed by employees.

Recently, less obvious techniques are being employed that involve ergonomic design and worker comfort. A comfortable employee, the theory maintains, can produce more than a counterpart who struggles through the day

Increases in productivity also can influence society more broadly, by improving living standards, and creating income.

When we measure productivity, we do not measure the quantity that was produced. We measure the relationship between input and output. If a business takes on more people to increase production, but the output per man hour drops to below the level when the extra people were hired, we cannot say that there was an increase in labour productivity.

The Inputs are:	The Outputs are:
Material s	Goods
Machines	Labour
Energy	Facilities
Technology.	Information
	Services

In business, inputs are measured in terms of costs and the organisation will strive to keep these costs as low as possible. Or the business will try to produce an output which is as high as possible, while keeping costs constant.

Put in another way, productivity is measured by how many products were produced (outputs) and what the cost was to produce the products by making use of inputs.

Measuring productivity

The productivity of any job can be measured in terms of:

- ✓ Money - costs and profits.
- ✓ Time - spent.
- ✓ Manpower - utilised.
- ✓ Materials - utilised.
- ✓ Products - quality and amount delivered.
- ✓ Service - quality and efficiency.

$$\text{Productivity} = \frac{\text{Product (output) quantity}}{\text{Resource (input) quantity}}$$

Productivity is a state of mind, it is when one is continuously improving, to perform better today than yesterday!

Commitment to good working practice

- ✓ I perform my duties and tasks to the best of my abilities
- ✓ I do everything to the best standard
- ✓ I am a model employee
- ✓ I set the example of how things should be done

- ✓ My good work makes me a great employee

The objective of a government department is primarily to render a service as opposed to making a profit. However, this does not mean that they are free to spend their funds to satisfy any or all the needs of the community. They have the following commitments:

- ✓ To determine the particular community needs which they wish to satisfy.
- ✓ To satisfy them in the most efficient manner.

This implies, that government must also give attention to the input-output relationship. To put it into other words, the government department must produce the highest possible output (need satisfaction) with the lowest possible input (resources).

This means that a government organisation must also make optimal use of the resources available to it.

Formative assessment

Discuss the following questions in a group and then note your answers:

In your own words describe why energy resources should be distributed equitably amongst the people and businesses in South Africa.
How can government ensure equitable distribution of water resources?
What happens if government does not render a waste disposal service?

How can government ensure equitable distribution of resources to encourage agricultural development?

Why should welfare benefits also be distributed equitably among the needy?

The laws regarding the selection and governing of national, provincial and local governments are set out in the Constitution. Section 214 of the Act states that the provinces are entitled to an equitable share of the national revenue. This includes a portion of the income tax on individuals collected in the province and a percentage of the value added or sales tax collected within the province. Explain how Section 214 of the Act ensures equitable distribution of resources.

The Constitution also sets out the obligations regarding providing of services by local governments. List at least four services that local governments must provide.
Should local governments be allowed to charge for providing these services? Motivate your answer.
In business productivity is the relationship between input and output. Describe how productivity is measured in government departments.
The objective of a government department is primarily to render a service as opposed to making a profit. However, this does not mean that they are free to spend their funds to satisfy any or all the needs of the community. They have the following commitments: list the

commitments.

Formative assessment

In a group, read the following article from 2006 abut provincial spending. Discuss the article in a group. Answer the following questions:

The following statement is made: "He said KZN in particular was under-spending on infrastructure". How would this affect the people living there regarding travel to and from work, health care, providing of schooling for children and access to water?
Mr Coetzee also said that salaries should represent only 40% of spending. How much did the provinces actually spend on current payments, which was largely made up of salaries?
How do you think this will affect the spending on other items such as health care, schools, water, roads, etc? Will there still be enough money left to improve these services, if the government officials spend most of the money on salaries?
Which province has spent the most on schools and colleges?

How much have the provinces spent on housing when compared to their budgets?
How much should they have spent?
By how much did KZN's spending on health budgets decrease?
Is this good or bad?
By how much did Gauteng's spending on health budgets increase?
Is this good or bad?
When you look at this article and the way provinces are spending the money allocated to them, do you think they are doing a good job?
What do you think the government should do about this?

PROVINCES TARDY ABOUT SPENDING: CLARE NAUDE

Posted: Tue, 01 Aug 2006 17:00 | © Moneyweb Holdings Limited, 1997-2006

THE nine provinces underspent their large budgets in the first quarter and capital spending in particular fell behind.

The recently published First Quarter Provincial Budget Report shows that the provinces have spent R38,4bn or 21% of the scheduled total of R183bn in the quarter to June. If they spent consistently, they would have paid out 25%.

Their spending is vital to retail sales especially in the less industrialised rural areas. Their total capital spending in the quarter came to a relatively paltry R2,44bn out of a planned R14,7bn – that was only 16,6% of the total.

Provincial spending has increased year-on-year by 8, 6% or R3bn.

Current payments, which represent largely salaries, represent the lion's share (77%) of provincial spending.

Clive Coetzee, of the Department of Economics at the University of KwaZulu-Natal (KZN) said ideally salaries should represent 40% of spending, transfers and social spending 30% and capex 30%.

He said KZN in particular was under-spending on infrastructure. Its roads, electricity, water, schools and hospitals all needed improvement but capex in the first quarter was only R561,7m out of a planned R3,5bn. Coetzee said there was no way growth could be achieved if consumption spending accounted for such a large part of provincial spending.

The total of all provinces' capital budget increased by 28,9% or R548, 8m more than the amount spent over the same period last year.

Only R479m out of a possible R17bn is spent over all the provinces on education capital expenditure. This seems extremely low when in order to grow we need to invest in capital for our foundations. But this is a 25,4% growth compared to last years spend, showing us that the incentive to boost growth is there.

These spending ranges, however, vary enormously between provinces. The Eastern Cape has spent heavily on schools and colleges. Its education capex rose 2019,6%, whereas KZN has seen an 80,6% decrease.

Health budgets are the second top spend totalling R10, 224bn for this quarter, giving a 15,8% rise compared to that spend of last year. Capital expenditure in this sector, however rose 64,3%, compared to that spending of last year. The nine provinces have currently spent 19,9% of their annual budget or R858, 3m.

Once again there are great variations between each province in this spending. KZN's spending decreased by 9,1% whereas Gauteng has increased by 376,4%.

Spending on housing and local government is low. Considering inflation over the past year has been around 4,8%, this sector has only grown by a mere 1,8% or R28, 3m. The provinces have also only spent 17% of the R9, 3bn budget.

Most of this housing and local government expenditure is on the Integrated Housing and Human Development conditional grant. With the Western Cape growing a huge 147, 6% compared to that of the Northern Cape, which has decreased 34,4%.

The biggest capital budgets in provinces are in public works, roads and transport departments at 35% of the total provincial capital budget of R14, 7bn. Spending for these departments is at 16,7% or R860, 8m that is 7,7% higher than the R799, 3m spent last year.

Gauteng has seen the largest increase of 383,1% whereas Mpumalanga has decreased by 46,2% compared to the spending this time last year