**LEARNER GUIDE**

Numeracy Level 2

**Unit Standard 9007 Level 2 Credits 5**

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**PERSONAL INFORMATION**

|  |  |
| --- | --- |
| ***NAME*** |  |
| ***CONTACT ADDRESS*** |  |
|  |
| ***Code*** |  |
| ***Telephone (H)*** |  |
| ***Telephone (W)*** |  |
| ***Cellular*** |  |
| ***Learner Number*** |  |
| ***Identity Number*** |  |
| ***EMPLOYER*** |  |
| ***EMPLOYER CONTACT ADDRESS*** |  |
|  |
| ***Code*** |  |
| ***Supervisor Name*** |  |
| ***Supervisor Contact Address*** |  |
|  |
| ***Code*** |  |
| ***Telephone (H)*** |  |
| ***Telephone (W)*** |  |
| ***Cellular*** |  |

**INTRODUCTION**

***Welcome to the learning programme***

Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have learnt. Take notes and share information with your colleagues. Important and relevant information and skills are transferred by sharing!



This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you!

## Structure

### Programme methodology



The programme methodology includes facilitator presentations, readings, individual activities, group discussions and skill application exercises.

**Know what you want to get out of the programme from the beginning and start applying your new skills immediately. Participate as much as possible so that the learning will be interactive and stimulating.**

The following principles were applied in designing the course:

* Because the course is designed to maximise interactive learning, you are encouraged and required to participate fully during the group exercises
* As a learner you will be presented with numerous problems and will be required to fully apply your mind to finding solutions to problems before being presented with the course presenter’s solutions to the problems
* Through participation and interaction the learners can learn as much from each other as they do from the course presenter
* Although learners attending the course may have varied degrees of experience in the subject matter, the course is designed to ensure that all delegates complete the course with the same level of understanding
* Because reflection forms an important component of adult learning, some learning resources will be followed by a self-assessment which is designed so that the learner will reflect on the material just completed.

This approach to course construction will ensure that learners first apply their minds to finding solutions to problems before the answers are provided, which will then maximise the learning process which is further strengthened by reflecting on the material covered by means of the self-assessments.

***Different role players in delivery process***

* Learner
* Facilitator
* Assessor
* Moderator

### What Learning Material you should have

This learning material has also been designed to provide the learner with a comprehensive reference guide. It is important that you take responsibility for your own learning process; this includes taking care of your learner material. You should at all times have the following material with you:

|  |  |
| --- | --- |
| ***Learner Guide*** | ***This learner guide is your valuable possession:***  This is your textbook and reference material, which provides you with all the information you will require to meet the exit level outcomes. During contact sessions, your facilitator will use this guide and will facilitate the learning process. During contact sessions a variety of activities will assist you to gain knowledge and skills.  Follow along in the guide as the training practitioner takes you through the material. Make notes and sketches that will help you to understand and remember what you have learnt. Take and share information with your colleagues. Important and relevant information and skills are transferred by sharing!  This learning programme is divided into sections. Each section is preceded by a description of the required outcomes and assessment criteria as contained in the unit standards specified by the South African Qualifications Authority. These descriptions will define what you have to know and be able to do in order to be awarded the credits attached to this learning programme. These credits are regarded as building blocks towards achieving a National Qualification upon successful assessment and can never be taken away from you! |
| ***Formative Assessment Workbook*** | The Formative Assessment Workbook supports the Learner Guide and assists you in applying what you have learnt.  The formative assessment workbook contains classroom activities that you have to complete in the classroom, during contact sessions either in groups or individually.  You are required to complete all activities in the Formative Assessment Workbook. The facilitator will assist, lead and coach you through the process. These activities ensure that you understand the content of the material and that you get an opportunity to test your understanding. |

### Different types of activities you can expect

To accommodate your learning preferences, a variety of different types of activities are included in the formative and summative assessments. They will assist you to achieve the outcomes (correct results) and should guide you through the learning process, making learning a positive and pleasant experience.



The table below provides you with more information related to the types of activities.

| ***Types of Activities*** | ***Description*** | ***Purpose*** |
| --- | --- | --- |
| ***Knowledge Activities*** | You are required to complete these activities on your own. | These activities normally test your understanding and ability to apply the information. |
| ***Skills Application Activities*** | You need to complete these activities in the workplace | These activities require you to apply the knowledge and skills gained in the workplace |
| ***Natural Occurring Evidence*** | You need to collect information and samples of documents from the workplace. | These activities ensure you get the opportunity to learn from experts in the industry.  Collecting examples demonstrates how to implement knowledge and skills in a practical way |

### Assessments

The only way to establish whether a learner is competent and has accomplished the specific outcomes is through the assessment process. Assessment involves collecting and interpreting evidence about the learners’ ability to perform a task.

**To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.**

**This programme has been aligned to registered unit standards. You will be assessed against the outcomes as stipulated in the unit standard by completing assessments and by compiling a portfolio of evidence that provides proof of your ability to apply the learning to your work situation.**



***How will Assessments commence?***

***Formative Assessments***

The assessment process is easy to follow. You will be guided by the Facilitator. Your responsibility is to complete all the activities in the Formative Assessment Workbook and submit it to your facilitator.

***Summative Assessments***

You will be required to complete a series of summative assessments. The Summative Assessment Guide will assist you in identifying the evidence required for final assessment purposes. You will be required to complete these activities on your own time, using real life projects in your workplace or business environment in preparing evidence for your Portfolio of Evidence. Your Facilitator will provide more details in this regard.

**To qualify and receive credits towards your qualification, a registered Assessor will conduct an evaluation and assessment of your portfolio of evidence and competency.**

### Learner Support

**The responsibility of learning rests with you, so be proactive and ask questions and seek assistance and help from your facilitator, if required.**



Please remember that this Skills Programme is based on outcomes based education principles which implies the following:

* You are responsible for your own learning – make sure you manage your study, research and workplace time effectively.
* Learning activities are learner driven – make sure you use the Learner Guide and Formative Assessment Workbook in the manner intended, and are familiar with the workplace requirements.
* The Facilitator is there to reasonably assist you during contact, practical and workplace time for this programme – make sure that you have his/her contact details.
* You are responsible for the safekeeping of your completed Formative Assessment Workbook and Workplace Guide
* If you need assistance please contact your facilitator who will gladly assist you.
* If you have any special needs please inform the facilitator

## Learner Administration



***Attendance Register***

You are required to sign the Attendance Register every day you attend training sessions facilitated by a facilitator.

***Programme Evaluation Form***

On completion you will be supplied with a “Learning programme Evaluation Form”. You are required to evaluate your experience in attending the programme.

Please complete the form at the end of the programme, as this will assist us in improving our service and programme material. Your assistance is highly appreciated.

### Learner Expectations

Please prepare the following information. You will then be asked to introduce yourself to the instructor as well as your fellow learners



|  |
| --- |
| Your name: |
|  |
|  |
| The organisation you represent: |
|  |
|  |
| Your position in organisation: |
|  |
|  |
| What do you hope to achieve by attending this course / what are your course expectations? |
|  |
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# UNIT STANDARD 9007

#### Unit Standard Title

Work with a range of patterns and functions and solve problems

#### NQF Level

2

#### Credits

5

#### Purpose

This unit standard is designed to provide credits towards the mathematical literacy requirements of the NQF at level 2. The essential purposes of the mathematical literacy requirements are that, as the learner progresses with confidence through the levels, the learner will grow in:

* An insightful use of mathematics in the management of the needs of everyday living to become a self-managing person
* An understanding of mathematical applications that provides insight into the learner’s present and future occupational experiences and so develop into a contributing worker
* The ability to voice a critical sensitivity to the role of mathematics in a democratic society and so become a participating citizen.

#### Learning Assumptions

The credit value is based on the assumption that people starting to learn towards this unit standard are competent in Mathematics and Communications at NQF level 1

#### Range

This unit standard includes the requirement to:

* Use algebraic notation to express generality
* Make conjectures, demonstrate and explain their validity
* Recognise equivalence among expressions and situations resulting from manipulation and rearrangement to forms appropriate for solving problems

Work with:

* Functions for which there are rules and for which there are no rules;
* Functions that are discrete (rules and no rules);
* Functions that are continuous (rules and no rules).

Investigate, and interpret graphs of situations with regard to the following: -increasing /decreasing,

* Maximal /minimal,
* Continuous / discrete,
* Rate of change,
* Intercepts,
* Interpolation /extrapolation.

(The above must be done in relation to the contexts in which the functions are acting as models.)

Work with the following basic functions: y =ax +b; y =LIX\*+b; y =ax; xy=k, In terms of their:

* Shape and symmetry,
* Finding function values,
* Finding input values,
* Analysing the behaviour of function values (the rate of change).

Represent, interpret and solve problems that relate to these functions by using point-by-point plotting and numerical analysis

Convert flexibly among various representations of the above functions (i.e. words, tables, formulae, graphs).

Learners are not expected to master each concept and procedure when they first encounter it, but rather to continually develop their mathematical understandings through encounters with mathematical models of realistic situations.

The contexts and situations should be used to develop a critical awareness of human rights, social, economic, political, cultural and environmental issues. Examples of the power of modelling as a descriptive tool to describe situations between two variables and as an analytic tool to gain additional information about the situation must be developed.

#### Specific Outcomes and Assessment Criteria

**Specific outcome 1:** Convert flexibly between and within various representations of functions

Range: This outcome includes the requirement to:

* Translate from one representation to another (i.e. verbal, tables, formulae, graphs).
* Deal with situations involving the range of functions specified in the main range statement as well as functions for which there is no rule

**Assessment criteria:**

* Appropriate information is selected to convert flexibly between and within various representations of functions.
* Appropriate representations are selected for specific applications.
* Conversions represent the functions accurately and appropriately.

**Specific outcome 2:** Compare, analyse and describe the behaviour of patterns and functions

Range: This outcome includes the requirement to work with functions:

* Identify, contrast and compare the features of the functions listed in the main range statement as well as functions for which there are no rules
* Recognise equivalent forms of an expression, equation or function

**Assessment criteria**

* Patterns and functions are compared in terms of: Shape and symmetry, Finding function values, Finding input values, The average rate of change of function values.
* The key features of the graphs of functions are described and interpreted correctly.
* The behaviour of functions is described as being increasing or decreasing or constant as determined visually from graphical representations.

**Specific outcome 3:** Represent situations mathematically in order to interpret and solve problems.

Range: This outcome includes the requirement to:

* Use expressions, functions and equations to represent situations
* Develop strategies for deciding whether symbolic, representations are reasonable and interpret such results

**Assessment criteria**

* Accurate point-by-point plotting is used to model contextual problems.
* Representations are analysed and manipulated efficiently in arriving at results.
* Representations are verified in terms of available data.
* Results are interpreted correctly in terms of the situation.
* Interpretations and predictions are based on the properties of the mathematical model.

#### Essential embedded knowledge

The following essential embedded knowledge will be assessed through assessment of the specific outcomes in terms of the stipulated assessment criteria. Candidates are unlikely to achieve all the specific outcomes, to the standards described in the assessment criteria, without knowledge of the listed embedded knowledge. This means that the possession or lack of the knowledge can be inferred directly from the quality of tile candidate’s performance against the standards.

* Relationships between variables
* Mathematical functions
* Representations of functions and relations

#### Critical cross field outcomes

* Identify and solve problems using critical and creative thinking: Solve a variety of problems based on patterns and functions
* Collect, analyse, organise and critically evaluate information: Gather, organise, evaluate and interpret information to compare and represent relationships and functions
* Communicate effectively: Use everyday language and mathematical language to describe relationships, processes and problem solving methods
* Use mathematics: Use mathematics to, describe and represent realistic and abstract situations and to solve problems.

# Representations of Functions

#### Outcome

Convert flexibly between and within various representations of functions

#### Outcome Range

This outcome includes the requirement to

* Translate from one representation to another (i.e. verbal, tables, formulae, graphs).
* Deal with situations involving the range of functions specified in the main range statement as well as functions for which there is no rule.

#### Assessment criteria

* Appropriate information is selected to convert flexibly between and within various representations of functions
* Appropriate representations are selected for specific applications
* Conversions represent the functions accurately and appropriately

## The language used in mathematics

### Equation

An equation is identified when a = sign is part of a sentence, for example 3 + 3 = 6. This is an important part of mathematics. From basics, problems are solved by the use of equations.

Multiplication, division, adding and subtraction, in the same expression: If the above operations are in the same expression, it is necessary to first multiply and divide and then add and subtract. However, work should always be carried out from left to right. For example:

a) 4+5-2x5

= 4+5‑10

= 9-10

=-1

## Activity 1

### The addition and subtraction of algebraic terms

It should be noted that only similar terms can be added or subtracted. Examples:

a) 7a + 3a ‑ 5a

(7 + 3 ‑ 5) a

5a

b) 4a + 4b + 2ab ‑ 7a + ab

4b + 4a ‑ 7a + 2ab + ab

4b + (4 ‑ 7)a + (2 + 1)(ab)

4b ‑ 3a + 3ab

c) Add 5a + 4ab ‑ 7b and 3 ‑ 3b + 5a

Solution:

5a + 4ab ‑ 7b

+5a ‑ 3b + 3

10a + 4ab ‑ 10b + 3

d) Subtract 4x + xy ‑ 2 from 5x + xy ‑3

5x + xy – 3

‑(4x + xy – 2)

X + 1

Always rearrange the terms to ensure that similar terms are positioned underneath each other.

## Activity 2

### Variables

Variables are very important in mathematics. They are used from quite early in a school career. For example 3 + # = 7.

The # represents an unknown which is called a variable.

The same expression can also be written as 3 + x = 7, where the expression x represents the variable.

If there are two unknowns in the equation, then the term variable is preferred. For example: x‑ y = 2.

From the above, it can be seen that unknowns can represent many values, hence the name "variables".

This equation is true if x and y represent the following values:

2 ‑ 0 = 2

3‑1 =2

4 ‑ 2 = 2

### Brackets

Brackets are placed in order to indicate multiplication, therefore

(a) = 2 x a

3(a) = 3 x a

(a)3 = a x a x a

Brackets provide assistance in clearly writing expressions. For example, if a + b is multiplied by 4 it is written as 4(a + b)

This is equal to3xa +3xb=3a+3b

If there is no number before a bracket, it is assumed that it is 1, for example:

+(a + b) = +1 (a + b), or ‑(a + b) = ‑1 (a + b) and (a + b) = 1(a + b) = a + b

#### Examples

Note, the x must be done first, and then after the bracket has been removed, similar terms must be added or subtracted

a) 4(3+x) – 2(7-x) = 12+4x – 14+2x

=6x-2

b) 4(x+2x) – 3x(2x-2x+3)

=4x+8x – 9x

=3x

c) 2(3a + 3) ‑ 4(a ‑ 8) 6a + 6 ‑ 4a + 32

2a + 38

Remember, that when multiplying signs, unlike signs give minus. Like signs give plus:

+X+ =+

-X- =+ +X- =-

### Multiplication, division, adding and subtraction, in the same expression

If the above operations are in the same expression, it is necessary to first multiply and divide and then add and subtract. However, work should always be carried out from left to right. For example:

a) 4+5‑2x5 2 = 4+5-2x

= 4+5-10

= 4+5-5

4

Exercises

a) 4 ‑ 6 + 10 5

b) 2+4x3 6x2‑3

Answers:

a) 0

b) 2

### The multiplication of variables

Time is wasted when a X a X a X a X a is written. However, this is necessary in mathematics, therefore a shorter notation was developed and a X a X a X a X a is written as a . Take note that there are four multiplication signs and not five. Thus a is multiplied four times by itself, but there are five a's in the expression

a5 is read as "a exponent 5", or "index 5" 5 is called the exponent or index a is called the base a is called "the power~'

#### Multiplication of similar terms

Examples:

a) 2x2 2 2x(2x2) = 23= 8

b) bxb b'

c) bxbxb = b'

d) a 2 x a 2 (a x a x a x a)= a 4

e) Xy X Xy (Xy)2

Therefore, if their bases are the same and the variables are multiplied, the exponents are added.

#### Multiplication if coefficients differ

Examples:

a) 2a x 3a (2 x 3)a"1

6a 2

b) 3k2 x 5x2 (3 x 5)(X2 X2 X2

1 5X2+2

#### Multiplication of terms where powers differ

Examples:

a) 2xx 3x2 (2 x 3)xl +2

6x

b) 4a x 4b x 3a 2 (4 x 4 x 3)

48 a 3 b

c) (2X)2 x 3xx 2)~ y 2 2 X2 x 3x x 2x2 y

(2 2 x 3 x 2)x2+'+2y

24 )~ y

### Formulae

A formula is a relationship between quantities, and they are handy ways of writing problems. These equations make it easier to solve mathematical problems, as shown in the following example:

#### Example:

Calculate the number that must be added to 18 in order to give an answer of 27. This is a simple equation which could be calculated mentally. However, its very simplicity allows it to be used as a simple example

Solution:

Let the unknown number be x.

Therefore: x + 18 = 27

Subtract 18 from both sides of the equation.

x + 18 ‑ 18 = 27 ‑ 18

Therefore: X = 9

In more complex problems there are more variables. The symbol that must be solved can be engaged in different ways to the formula. The most general operations that engage variables are [x], and [‑], and powers.

The following examples show how the above mentioned operations engage y.

a) y + 4 = 28

b) y ‑ 4 = 28

c) y = 28 x 3

d) 4y = 28

e) y³ = 28

It should be noted that if something is added to y and it is necessary to solve for y in the opposite direction, then subtraction is applied.

**Combinations of + and** ‑

**Example:**

Solve for y if y + a ~ 5 = 9

Solution:

y + a ‑ 5 = 9

Therefore: y + a ‑ 5 ‑ a + 5 = 9 ‑ a + 5

Therefore: y = 14 ‑ a

**Combinations of x and ÷** ‑Example:

'Solve for y if 4X = 32

Solution:

4xX = 32

Therefore: X = 32÷4

Therefore: X = 6

**Combinations of x and +**

Example: 3Y + a = 6 divide each term by 3

Solution: 3Y +a= 6

Therefore: Y+a/3 =2

Therefore: Y + a /3‑ a/3 = 2 – a/3

Therefore: Y = 2 – a/3

The above also applies for combinations of x and -

Combinations of ‑L, + and ‑

**Example:**

y +a -3 = 5

2

subtract a and add 3 to both sides of the equation.

y + a ‑ 3 ‑ a + 3 = 5 - a + 3

2

Therefore: y =8‑a

2

multiply by 2

y x 2 = (8‑ a)2

2.

Therefore: y =2(8‑a) and y =16‑2a

**Combination of powers, x and ‑.**

Example:

3 X³ = 40,5

2

multiply both sides of the equation by 2

Therefore: 3 X³ x 2 = 40,5 x 2

2

Therefore X³ = 81

3

X³ = 27

and x = 3

**Combination of powers, + and** ‑

**Example:**

X² +Y‑6=10

subtract y and add 6

Therefore: X² + y ‑ 6 ‑ y + 6 = 10 ‑ y + 6

Therefore: X² = 16 ‑ y

take √ on both sides of the equation.

X = √(16‑y)

Therefore: x= 16‑y

The right hand side of the equation can be simplified, but not the left hand side.

Expressions containing = and ‑ cannot be easily simplified.

Combinations of powers, +, ‑, X, and ‑

**Example:**

4/3a²‑ 3 + b = 2

Therefore: 4/3 a² x3/4‑3x3/4+bx3/4=2x3/4

Therefore: a²‑9/4 + 3/4b =6/4

Therefore: a²‑9/4 +3/4b +9/4 ‑3/4b =6/4 + 9/4 ‑3/4b

Therefore: a²= 15/4 – 3/4b

## Functions

The mathematical concept of a function expresses dependence between two quantities, one of which is given (the independent variable, argument of the function, or its "input") and the other produced (the dependent variable, value of the function, or "output").

A function associates a single output to each input element drawn from a fixed set, such as the real numbers.

There are many ways to give a function: by a formula, by a plot or graph, by an algorithm that computes it, or by a description of its properties.

In applied disciplines, functions are frequently specified by their tables of values or by a formula. Not all types of description can be given for every possible function, and one must make a firm distinction between the function itself and multiple ways of presenting or visualising it.

One idea of enormous importance in all of mathematics is composition of functions:

if z is a function of y

and y is a function of x,

then z is a function of x.

We may describe it informally by saying that the composite function is obtained by using the output of the first function as the input of the second one.

This feature of functions distinguishes them from other mathematical constructs, such as numbers or figures, and provides the theory of functions with its most powerful structure.

Functions in algebra are usually expressible in terms of algebraic operations. The term transformation is often synonymous with function. The term transformation usually applies to functions whose inputs and outputs are elements of the same set or more general structure. Thus, we speak of linear transformations from a vector space into itself and of symmetry transformations of a geometric object or a pattern.

Mathematical functions are denoted frequently by letters, and the standard notation for the output of a function ƒ with the input *x* is ƒ(*x*). A function may be defined only for certain inputs, and the collection of all acceptable inputs of the function is called its domain. The set of all resulting outputs is called the range of the function

It is a usual practice in mathematics to introduce functions with temporary names like ƒ; in the next paragraph we might define ƒ(x) = 2x+1, and then ƒ(3) = 7. When a name for the function is not needed, often the form y = x2 is used.

If we use a function often, we may give it a more permanent name as, for example,

\operatorname{Square}(x) = x^2 . \,\!

The essential property of a function is that for each input there must be a unique output. Thus, for example, the formula

\operatorname{Root}(x) = \pm \sqrt x 

does not define a real function of a positive real variable, because it assigns two outputs to each number: the square roots of 9 are 3 and −3. To make the square root a real function, we must specify, which square root to choose. The definition

\operatorname{Posroot}(x) = \sqrt x \,\!

for any positive input chooses the positive square root as an output.

### Words

A function need not involve numbers. By way of examples, we find

* the function that associates with each word its first letter
* or the function that associates with each triangle its area.

Because functions are used in so many areas of mathematics, and in so many different ways, no single definition of function has been universally adopted

### Definition

One simple intuitive definition, for functions on numbers, says:

A function is given by an arithmetic expression describing how one number depends on another.

An example of such a function is y = 5x−20x3+16x5, where the value of y depends on the value of x.

This is entirely satisfactory for parts of elementary mathematics, but is too clumsy and restrictive for more advanced areas.

### Sets

Eventually the gradual transformation of intuitive "calculus" into formal "analysis" brought the need for a broader definition. The emphasis shifted from how a function was presented — as a formula or rule — to a more abstract concept. Part of the new foundation was the use of sets, so that functions were no longer restricted to numbers. Thus we can say that

A function ƒ from a set X to a set Y associates to each element x in X an element y = ƒ(x) in Y.

Note that X and Y need not be different sets; it is possible to have a function from a set to itself. Although it is possible to interpret the term "associates" in this definition with a concrete rule for the association, it is essential to move beyond that restriction.

For example, we can sometimes prove that a function with certain properties exists, yet not be able to give any explicit rule for the association. In fact, in some cases it is impossible to give an explicit rule producing a specific y for each x, even though such a function exists.

### Partial function

As functions take on new roles and find new uses, the relationship of the function to the sets requires more precision. Perhaps every element in *Y* is associated with some *x*, perhaps not.

In some parts of mathematics it is convenient to allow values of x with no association (in this case, the term partial function is often used). To be able to discuss such distinctions, many authors split a function into three parts, each a set:

* A function ƒ is an ordered triple of sets (*F*,*X*,*Y*) with restrictions, where

*F* (the **graph**) is a set of ordered pairs (*x*,*y*),

*X* (the **source**) contains all the first elements of *F* and perhaps more, and

*Y* (the **target**) contains all the second elements of *F* and perhaps more.

The most common restrictions are that F pairs each x with just one y, and that X is just the set of first elements of F and no more.

### Range

The range of F, and of ƒ, is the set of all second elements of F; it is often denoted by rng ƒ. The domain of F is the set of all first elements of F; it is often denoted by dom ƒ. There are two common definitions for the domain of ƒ. Some authors define it as the domain of F, while others define it as the source of F.

The target Y of ƒ is also called the codomain of ƒ, denoted by cod ƒ; and the range of ƒ is also called the image of ƒ, denoted by im ƒ. The notation ƒ:X→Y indicates that ƒ is a function with domain X and codomain Y.

### Argument

A specific input in a function is called an argument of the function. For each argument value x, the corresponding unique y in the codomain is called the function value at x, or the image of x under ƒ. The image of x may be written as ƒ(x) or as y.

### Graph

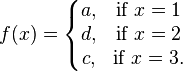
The graph of a function ƒ is the set of all ordered pairs (x, ƒ(x)), for all x in the domain X. If X and Y are subsets of R, the real numbers, then this definition coincides with the familiar sense of "graph" as a picture or plot of the function, with the ordered pairs being the Cartesian coordinates of points.

The concept of the image can be extended from the image of a point to the image of a set. If A is any subset of the domain, then ƒ(A) is the subset of the range consisting of all images of elements of A. We say the ƒ(A) is the image of A under f.

Notice that the range of ƒ is the image ƒ(X) of its domain, and that the range of ƒ is a subset of its codomain.

In mathematics, the graph of a function f is the collection of all ordered pairs (x,f(x)). In particular, graph means the graphical representation of this collection, in the form of a curve or surface, together with axes, etc. Graphing on a Cartesian plane is sometimes referred to as curve sketching.

The graph of the function

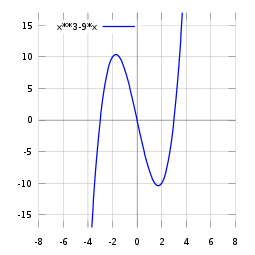


is {(1,a), (2,d), (3,c)}.

The graph of the cubic polynomial on the real line

f(x)={{x^3}-9x} \!\ 

is {(x,x3-9x) : x is a real number}. If the set is plotted on a Cartesian plane, the result is

[](http://en.wikipedia.org/wiki/Image:Cubicpoly.svg)

### Specifying a function

A function can be defined by any mathematical condition relating each argument to the corresponding output value. If the domain is finite, a function ƒ may be defined by simply tabulating all the arguments x and their corresponding function values ƒ(x). More commonly, a function is defined by a formula, or (more generally) an algorithm — a recipe that tells how to compute the value of ƒ(x) given any x in the domain.

# Behaviour of Patterns and Functions

#### Outcomes

Compare, analyse and describe the behaviour of patterns and functions

Represent situations mathematically in order to interpret and solve problems

#### Outcome Range

This outcome includes the requirement to work with functions

* Identify, contrast and compare the features of the functions listed in the main range statement as well as functions for which there are no rules
* Recognise equivalent forms of an expression, equation or function

#### Assessment criteria

* Patterns and functions are compared in terms of: Shape and symmetry, finding function values, iii. Finding input values, the average rate of change of function values.
* The key features of the graphs of functions are described and interpreted correctly
* The behaviour of functions is described as being increasing or decreasing or constant as determined visually from graphical representations
* Accurate point-by-point plotting is used to model contextual problems
* Appropriate symbolic representations are used to model contextual problems
* Representations are analysed and manipulated efficiently in arriving at results.
* Representations are verified in terms of available data
* Results are interpreted correctly in terms of the situation
* Interpretations and predictions are based on the properties of the mathematical model

The system of Cartesian coordinates is the most commonly used coordinate system. In two dimensions, this system consists of a pair of lines on a flat surface (or plane) that intersect at right angles. Each of the lines is called an axis and the point at which they intersect is called the origin. The axes are usually drawn horizontally and vertically and are usually referred to as the x and y axes, respectively.

In Cartesian coordinates, a point on the plane whose coordinates are (2,3) is 2 units to the right of the y axis and 3 units up from the x axis. The figure on the next page show four points (2,3), (-2.3,4.5), (-4,-3) and (3,-4).

Cartesian

Cartesian or rectangular coordinates

The four points occupy different quadrants of the graph, I, II, III and IV. By convention, the coordinates start in the upper right hand corner and count counter clockwise. In addition, each quadrant is represented by the capital roman numerals for 1, 2, 3 and 4 respectively.

The system of latitude and longitude is another example of a coordinate system that uses two coordinates to specify the position of a point on the surface of the earth. The coordinate system for latitude and longitude is spherical and not rectangular because the earth is a sphere and not a cube.

When we draw a map, we represent towns as points in the plane (map). If we wish to describe the position of a town B with respect to town A, we draw the perpendicular North-South ad East-West lines at town A (0).

W

E

N

S

45˚

A

B

10km

We can now see these lines as the axes of the Cartesian plane. Any point on this plane can be described by (x;y) where x is its position in terms of the X-axis and y its position in terms of the Y axis. X and y are called the coordinates of the point.

4

3

2

1

-1

-2

-3

-4

1

2

3

4

-1

-2

-3

-4

y

x

x

y

(-2,-2)

(1,1)

(3,1)

One can use now the theorem of Pythagoras to calculate distances between points on the Cartesian plane. Look again at the first figure. What are the coordinates of town B?

**The theorem of Pythagoras states the following**:

In any right-angled triangle the following is true:

The square of the hypotenuse is equal to the sum of the squares of the other two sides (AC) = (AB)+ (BC)

C

A

B

According to this theorem one possibility for the coordinates of town B in figure 1 is (4, 3) since 4+ 3= 5