

**Working With Numbers in Various Contexts**

**US No: 7447 Level 1, Credits 6**

**LEARNER MANUAL**

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| --- | --- |
| Learner’s name |  |
| Facilitator’s name |  |
| Starting date |  |

**Before we start…**

Dear Learner - on completion of this Learner Guide, you will have acquired all the knowledge and skills to be assessed against the following unit standard:

Title: Working with numbers in various contexts

US No: 7447 NQF Level: 1 Credits: 6

The full unit standard is attached at the end of this module. Please read the unit standard at your own time. Whilst reading the unit standard, make a note of your questions and aspects that you do not understand, and discuss it with your facilitator.

You will also be handed a Learner Workbook. This Learner Workbook should be used in conjunction with this Learner Guide. The Learner Workbook contains the activities that you will be expected to do during the course of your study. Please keep the activities that you have completed as part of your Portfolio of Evidence, which will be required during your final assessment.

You will be assessed during the course of your study. This is called formative assessment. You will also be assessed on completion of this unit standard. This is called summative assessment. Before your assessment, your assessor will discuss the unit standard with you.

**How to use this guide …**

Throughout this guide, you will come across certain re-occurring “boxes”. These boxes each represent a certain aspect of the learning process, containing information, which would help you with the identification and understanding of these aspects. The following is a list of these boxes and what they represent:

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| **Definition** | **What does it mean?** Each learning field is characterized by unique terms and **definitions** – it is important to know and use these terms and definitions correctly. These terms and definitions are highlighted throughout the guide in this manner. |

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| **Activity** | You will be requested to complete **activities,** which could be group activities, or individual activities. Please remember to complete the activities, as the facilitator will assess it and these will become part of your portfolio of evidence. Activities, whether group or individual activities, will be described in this box. |

|  |  |
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| **Example** | Examples of certain concepts or principles to help you contextualise them easier, will be shown in this box. |

**What are we going to learn?**

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| --- | --- | --- |
| **Section** | **Contents** | **Page**  **No** |
|  | **What will I be able to do? (Learning Outcomes)** | **3** |
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| **1** | **Counting Systems From Different Cultures**  Our own; Egyptian; Roman; Different number base systems. | **6** |
| **2** | **Development of the Base-ten Number System**  Development and significance of zero; Place value of numbers; Patterned nature of whole numbers; The decimal number system. | **14** |
| **3** | **3: Rational and Whole Numbers and**  Different types of numbers; The properties of whole numbers, rational numbers and integers; The difference between rational and whole numbers; The increasing density of each type of numbers; Whole numbers as a subset of rational numbers. | **21** |
| **4** | **Mathematical Symbols and Numerical Models** Mathematical sentences; Some everyday problems; Explore some numerical models (equations, expressions and terms); Apply numerical models (meaning of and relationships between symbols). | **24** |
| **5** | **Solve Everyday Problems Using Estimation & Calculations.**  Mathematical and number problem solving strategies; Apply problem solving strategies correctly; Estimate first; Importance to calculate accurately; Calculation must follow some form of logical reasoning process; Check our solutions. | **27** |
| **6** | **Solutions Within Different Contexts**  Check and verify our own solutions and that of others; Explaining the reasoning process clearly; Justifying solutions in terms of the context; Solutions are shown to be consistent with estimations and vice versa. | **32** |
| **7** | **Simple and Complex Numerical Expressions**  The conventions governing the order of operations; Four basic operations in all combinations; Number operations without a calculator; Calculate expressions involving exponents without a calculator; Perform number operations with a calculator; Check that your solutions are correct. | **34** |

**What will I be able to do? (Learning Outcomes)**

**When you have achieved this unit standard, you will be able to:**

* Express and interpret a range of contexts using mathematical symbols, and find applications for numerical models;
* Solve a range of everyday problems using estimation and calculations
* Verify and justify solutions within different contexts;
* Perform operations on simple and complex numerical expressions;
* Describe and compare counting systems from different cultures;
* Critically analyse the development of the base ten number system;
* Apply the relationship between rational and whole numbers;
* Apply the relationship between rational numbers and integers.

**What do I need to know?**

The following competencies at ABET level 3 Numeracy are assumed to be in place:

* Solve realistic and abstract problems involving changing quantities by addition, subtraction, multiplication and division;
* Solve realistic and abstract problems involving variables in non-symbolic form;
* Demonstrate knowledge of different ways of expressing fractions and work with fractions, percentages and decimals to describe situations and calculate change situations;
* Demonstrate knowledge of the development of mathematics as a human activity and use alternate number system to the base ten system.

**Why is it Important to be Competent in Numeracy and Literate in Mathematics?**

**For once let’s start our learning experience by doing a little activity!**

Read the following article and then answer the questions that follow to dispel your dreads about doing Mathematics!

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| **Activity** | **Attitudes and Misconceptions**  Do your experiences in math cause you anxiety? Have you been left with the impression that math is difficult and only some people are 'good' at math? Are you one of those people who believe that you 'can't do math', that you're missing that 'math gene'?  Do you have the dreaded disease called Math Anxiety? Read on, sometimes our school experiences leave us with the wrong impression about math. There are many misconceptions that lead one to believe that only some individuals can do math. It's time to dispel those common myths… |

**Tick off true or false?**

|  |  |  |
| --- | --- | --- |
| Statement | False | True |
| 1. There is only one way to solve a problem… |  |  |
| 2. You need a 'Mathematics gene' or dominance of your left-brain to be successful at Mathematics… |  |  |
| 3. People don't learn the basics anymore because of a reliance on calculators and computers…. |  |  |
| 4. You need to memorize a lot of facts, rules and formulas to be good at Mathematics… |  |  |

Compare your answers:

**1. False.** *Why?* There are a variety of ways to solve mathematic problems and a variety of tools to assist with the process.

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| **Let’s test it for ourselves!**  **Think of the following problem…**  You bought a take-away pizza to share between yourself and five other friends. It has 8 slices. How would you divide 8 slices equally between 6 people? **Write down your method…**    \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  **Now compare your method with the person next to you and other learners in the class. Did you all use the same method?**  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Some of you will visualize the pizza, some will add the total number of slices and divide by 6. Does anyone actually write the algorithm? Not likely! There are a variety of ways to arrive at the solution, **and everyone uses their own learning style when solving the problem.** |

**2. False.** *Why?* Like reading, the majority of people are born with the ability to do mathematics. Children and adults need to maintain a positive attitude and the belief that they can do mathematics. This self-belief has often been scarred somewhere in the past… today is the day to make a fresh start and begin from scratch!

**3. False**. *Why?* Research at this time indicates that calculators do not have a negative impact on achievement. The calculator is a powerful teaching tool when used appropriately. Most facilitators now help you to learn how to use any technological tool to your advantage!

4**. False**. *Why*? As stated earlier, there's more than one way to solve a problem. Memorizing procedures is not as effective as conceptually understanding concepts! The question to ask yourself is: Do I really understand how, why, when this will work?

***Positive attitudes towards Mathematics are the first step to success!***

* **When does the most powerful learning usually occur?**
* If you take the time to analyse where you go wrong, you can't help but learn. Never feel badly about making mistakes in mathematics!
* Mathematics has never been more important, technology demands that we work smarter and have stronger problem solving skills!

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| **My Notes …**  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| **Session 1: Counting Systems From Different Cultures** |

After completing this session, you will be able to**: (SO 5) Describe and compare counting systems from different cultures.**

In this session we are going to explore the following concepts:

* **Counting systems from different cultures**
  + Our own - How this counting system developed and its significance
  + How this counting system developed and its significance.
  + Examples of how the systems might have been used.
  + The limitations of the system.
* **Egyptian - How this counting system developed and its significance**
  + How this counting system developed and its significance.
  + Examples of how the systems might have been used.
  + The limitations of the system.
* **Roman - How this counting system developed and its significance**
* How this counting system developed and its significance.
* Examples of how the systems might have been used.
* The limitations of the system.
* **Different number base systems, what they are used for and how to translate between them:**
  + Base 2
  + Base 5
  + Base 10
  + Base 16

|  |  |
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| **Definition** | **What does it mean?**  A **numera**l is a symbol or group of symbols that represents a number. Numerals differ from numbers just as words differ from the things they refer to. The symbols "11", "eleven" and "XI" are different numerals, all representing the same number. This article attempts to explain the various systems of numerals.  A **numeral system** (or **system of numeration**) is a framework where a set of numbers is represented by numerals in a consistent manner. It can be seen as the context that allows the numeral "11" to be interpreted as the binary numeral for three, the decimal numeral for eleven, or other numbers in a different bases. |

**We will now examine a number of counting systems, how the relevant counting system developed and its significance.**

## **Counting Systems from Different Cultures**

The natural numbers presumably had their origins in the words used to count things, beginning with the number one.

* **Use of Numerals**

The first major advance in abstraction was the use of numerals to represent numbers. This allowed systems to be developed for recording large numbers.

**For example:**

The **Babylonians** developed a powerful place-value system based essentially on the numerals for 1 and 10. The ancient **Egyptians** had a system of numerals with distinct hieroglyphs for 1, 10, and all the powers of 10 up to one million. A stone carving from **Karnak**, dating from around 1500 BC and now at the Louvre in Paris, depicts 276 as 2 hundreds, 7 tens, and 6 ones; and similarly for the number 4,622.

* **The idea of ZERO as a Numeral**

A much later advance in abstraction was the development of the idea of zero as a number with its own numeral. A zero digit had been used in place-value notation as early as 700 BC by the Babylonians, but it was never used as a final element. The Olmec and Maya civilization used zero as a separate number as early as 1st century BC, apparently developed independently, but this usage did not spread beyond Mesoamerica. The concept as used in modern times originated with the Indian mathematician Brahmagupta in 628 AD. Nevertheless, zero was used as a number by all medieval computists (calculators of Easter) beginning with Dionysius Exiguous in 525, but in general no Roman numeral was used to write it. Instead, the Latin word for "nothing," nullae, was employed.

* **Study of Numbers as Abstractions**

The first systematic study of numbers as abstractions (that is, as abstract entities) is usually credited to the Greek philosophers Pythagoras and Archimedes. However, independent studies also occurred at around the same time in India, China, and Mesoamerica.

* **Set-theoretical definition of natural Numbers**

In the nineteenth century, a set-theoretical definition of natural numbers was developed. With this definition, it was more convenient to include zero (corresponding to the empty set) as a natural number. This convention is followed by set theorists, logicians, and computer scientists. Other mathematicians, primarily number theorists, often prefer to follow the older tradition and exclude zero from the natural numbers.

## **Our Own Counting System**

The ancient Egyptians were using special symbols, known as pictographs, to write down numbers over 3,000 years ago. Later, the Romans developed a system of numerals that used letters from their alphabet rather than special symbols. Today, we use numbers based on the Hindu-Arabic system. We can write down any number using combinations of up to 10 different symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). The ancient Egyptians developed number systems to keep accounts of what was bought and sold.

Thousands of years ago there were no number digits for the concepts “two” or “three”. In place of these digits, fingers, toes, stones, sticks or even eyes were used to represent different numbers. There were no watches, calendars or any other time measuring devices available by then.

The **Arabic numerals**, 0, 1, 9, now in general use, are derived from Indian numerals. The name Arabic is used because Western Europeans learned about the system from Arabic writers. In Sanskrit literature number words for 1-9, 10, 100 and further powers of 10 - up to 10 - were used (similar to decimal system). The most widely used place value symbols belong to the Nagari script numerals, very similar to the Brahmi numerals, which form the basis of the modern Arabic numerals.

**Historians trace many modern numerals to the Brahmi numerals**, which were in use around the middle of the third century BC. The place value system, however, evolved later. Dating these numerals tells us that they were in use over quite a long time span up to the 4th century AD.

During the Gupta period (early 4th century AD to the late 6th century AD), the Gupta numerals developed from the Brahmi numerals and were spread over large areas by the Gupta empire as they conquered territory. Beginning around 7th century, the Gupta numerals evolved into the Nagari numerals.

**1.3 The Egyptian Counting System**

In Ancient Egypt, a need developed when people started living together in organised tribes in specific places. Out of this development, the need for barter systems grew, and thus the need for money based counting systems. The challenge was how to distinguish between five and fifty when the only concept of counting that you have in your vocabulary, are “herds”, “heaps” and “many”.

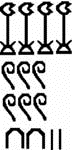
Paper and pencils to write down numbers, did not exist. They developed other methods to communicate in writing and to teach the other people around them the use of number systems.

The Egyptians had three systems of writing, the hieroglyphic, the hieratic, and the demotic writings. The hieroglyphic system was the sacred writing reserved for formal inscriptions and was usually used for writing on stone. Numbers were seldom used in this system. Hieroglyphics were too complex for everyday use and this led to the development of a simpler system, hieratic or temple writing. This was used by priests and scribes for everyday records and was mostly used for writing on papyrus. Demotic writing developed from hieratic writing and was the system of writing used in everyday life.

Egyptian numbers were written from right to left. They started at one and went up to a million. The numbers one to nine were written as combinations of vertical strokes, ten was represented by a sign that looks like an upturned U, 100 by a coil of rope, 1000 by a sign representing a lotus flower, 10,000 by a vertical finger, 100,000 by a tadpole, and 1 million by a man with upraised arms.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Value | 1 | 10 | 100 | 1,000 | 10,000 | 100,000 | 1 million, or  infinity |
| Hieroglyph | **l** |  |  |  |  | or |  |
| Description | Single  stroke | Cattle  hobble  or yoke | Coil of rope | Water lily  (also called  Lotus) | Finger | Tadpole  or Frog | Tadpole  or Frog |

Multiples of these values were expressed by repeating the symbol as many times as needed. For instance, a stone carving from Karnak shows the number 4622 as indicated on the right. Egyptian hieroglyphs could be written in both directions (and even vertically). This example is written left-to-right and top-down. On the original stone carving, it is right-to-left, and the signs are thus inverted.



* **The Limitations of the System**

There are two important limitations to such a system:

* Firstly, every different type of good for which you want to make a record must have its own distinctive sign. We saw how the increasing complexity of economic life led to a great increase of styles of tokens. Each of these tokens now had to be rendered by its own sign, and, of course, all the signs had to be learned.
* Secondly, the limitation concerns not the range of goods available, but their quantity. Recording a delivery or disbursement of three jars of oil by writing the oil-jar symbol three times is simple and convenient. Recording a delivery or disbursement of several hundred jars of oil the same way is no longer so convenient and is also a system to prone to error.

**1.4 The Roman Counting System**

Roman numerals are among the most enduring of number systems. Indeed, they are still in use today for certain purposes, such as numbering appendices in books, denoting dates in the credits of films and television programmes, and on watches and clocks. The long usage of Roman numerals could be due to a number of factors, including the widespread influence of the Roman Empire, the force of tradition, and the fact that the system had many advantages over other European systems of the time. For example, Roman numbers had the advantage that the majority of users had to memorize only a few symbols and their values.

The origin of the Roman number system is still obscure. One theory is that it may have been based on the number 5. Thus, the symbol for 5, V, could have its origins in a representation of a hand held flat with the fingers together, and X, for 10, could be a double V. Three of the other symbols could have come from Greek letters not needed in the early Roman alphabet: C (100) from theta (q), M (1000) from phi (f), and L (50) from chi (c, also written as ^). Q and f were probably changed gradually, under the influence of the initials of the number words centum (100), and mille (1000), to C and M.

The modern system of Roman numerals uses a subtractive principle. For example, 4 is written as IV, five minus one, rather than IIII, 9 is written as IX, ten minus one, rather than VIIII, and 900 is written as CM, 1000 minus 100, rather than DCCCC. Thus, for example, 1996 is written as MCMXCVI.

The system used in antiquity was slightly modified in the Middle Ages to produce the system we use today. It is based on certain letters that are given values as numerals:

**I or i for one,**

**V or v for five,**

**X or x for ten,**

**L or l for fifty,**

**C or c for one hundred (centum),**

**D or d for five hundred,**

**M or m for one thousand (mille).**

For the numbers not assigned a specific symbol, the above given symbols are combined:

**II or ii for two,**

**III or iii for three,**

**IV or iv for four,**

**VI or vi for six.**

For very large numbers (five thousand and above), a bar is placed above a base numeral to indicate multiplication by 1000;

**V for five thousand**

**X for ten thousand**

**L for fifty thousand**

**C for one hundred thousand**

**D for five hundred thousand**

**M for one million**

* **Examples of how the systems might have been used**

The Romans were active in trade and commerce, and from the time of learning to write they needed a way to indicate numbers. The system they developed lasted many centuries, and still sees some specialized use today.

Roman numerals traditionally indicate the order of rulers or ships that share the same name (i.e. Queen Elizabeth II). They are also sometimes still used in the publishing industry for copyright dates, and on cornerstones and gravestones when the owner of a building or the family of the deceased wishes to create an impression of classical dignity. The Roman numbering system also lives on in our languages, which still use Latin word roots to express numerical ideas. A few examples: unilateral, duo, quadriceps, septuagenarian, decade, and millilitre.

* **The Limitations of the System**

The big differences between Roman and Arabic numerals (the ones we use today) are that Romans didn't have a symbol for zero, and that numeral placement within a number can sometimes indicate subtraction rather than addition.

**1.5 Different Number Base Systems**

In this section, we will focus on different number base systems, what they are used for and how to translate between them:

* **Base 2**

This base-2 system (binary) is the **basis for digital computers**.

Switches, mimicked in their electronic successors built of vacuum tubes, have only two possible states: "open" and "closed". Substituting open=1 and closed=0 (or the other way around) yields the entire set of binary digits. It is used to perform integer arithmetic in almost all digital computers, with exceptions in the exotic base-3 (ternary) and base-10 designs that were discarded early in the history of computing hardware. Modern computers use transistors that have binary state as either high or low voltages. A computer does not treat all of its data as numerical. For instance, some of it may be treated as program instructions or data such as text. However, arithmetic and Boolean logic constitute a great part of operation. Real numbers, allowing fractional values, are usually approximated as floating point numbers which have different methods of arithmetic from integers.

* Digits used are the Indian/Arabic digits of 0 and 1. Each number occupies a place value. When 1 is reached, the value goes to 0 and 1 is added to the next place value: 0,1,10,11,100,101,110,111,1000, etc.
* Each place value to the left is equal to 2 times the place value to the right which implies that each place value to the right is equal to the place value to the left divided by 2.
* Continuing infinitely <- 256,128,64,32,16,8,4,2,1
* The use of 2 digits for a numbering system may be seen to arise from 3 sources.
* Ancient Chinese worldview. The development of two forces of energy continually ebbing, flowing, acting and reacting is embodied with the principles of yin and yang. Yang is represented by a solid line and yin by an open line.

These lines are then combined with each other twice to represent 4 phases of energy, strong yang, lesser yang, lesser yin and strong yin.

Then a third line is combined for more precision to represent a trigram of lines, giving a total of 8 possible trigrams representing 8 phases of energy, heaven (complete yang), thunder (strong yang), water (middle yang), mountain (lesser yang), earth (complete yin), wind (strong yin), fire (middle yin), lake (lesser yin).

Two trigrams are then combined for yet more precision to give the possibility of 64 hexagrams. This is not a formalized numbering system, but the binary logic and counting from 1,2,4,8 to the trigrams and then 8\*8 for the two trigrams making 64 hexagrams is evident.

Development of a logical system. The two logic states of true and false were developed into a more formalized numbering system.

Development of computerized logic. The two electrical states of - and + charge readily translated into a logic state of true and false. Each state or bit of information successively combined to give rise to nibbles (4 bits), bytes (8 bits) and more bits (depending on the model being used) to represent extremely large numbers, representations of letters, pictures, etc.

* **Base 5**

**Used primarily before the writing of numbers**. The signs or words used are hand for 5, two hands for 10, person for 20 (two hands and two feet). Some cultures would count on fingers with 0 being a closed fist and putting a finger up for one, etc., and some cultures would have 0 being an open hand and 1 would be signified by putting a digit down with 5 represented by a closed fist or 5 down.

Base 5 was used not as a formalized place value, but rather as a grouping value that combined with other values to a larger grouping value. For example 2 fives (two hands) are 10, 4 fives (two hands and two feet-or person) are 20. 12 fives are 60. Each of the groups 10, 20 and 60 were developed into more rigorous numbering systems throughout our globe's cultural history.

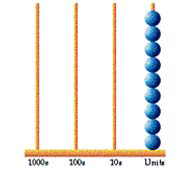
The use of 5 as a grouping value is easily understood as the hand has 5 digits.

This grouping was used for several hundreds of thousands of years by many cultures around our globe.

* **Base 10**

The base-10 system (decimal) is the one most commonly used today. It is assumed to have originated because humans have ten fingers.

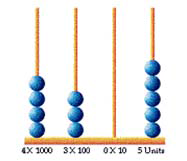
The most important feature of the Arabic number system is the use of place-value notation. Perhaps the best way to see how this works is to imagine that counting is carried out by threading beads onto wires arranged as in Diagram 1.



*Diagram 1: A Representation of*

*The Place-value Notation*

As each object is counted it is represented by a single bead threaded on the right-hand end wire. The wire has only enough room for nine beads. To count the tenth object, one bead is put on the next wire and all beads are taken off the end wire. Then the beads are used to fill up the end wire again, and so on. Thus each bead on the second wire represents ten; on the third wire one hundred; on the fourth wire one thousand, and so on. To write this down, all that are needed are nine symbols to represent from one to nine beads (in this case, the Arabic numerals 1 to 9) and one symbol (in this case, 0) to represent an empty wire. The symbols are written out in the same places as the wires: the value of a numeral depends on its place. For example, in Diagram 2, we can see that 4305 is a way of representing the number that consists of 4 thousands, 3 hundreds, 0 tens and 5 units or ones.



*Diagram 2*

Ten is called the base, or radix, of this system, which is the one now in everyday use. A notation system with ten as the base is called a denary system. Any other number could be used as a base.

Place-value notation is very difficult without a symbol for zero. If there were no zero symbol in the denary system then 9 could mean nine, ninety, nine hundred, etc. Zero may be a late invention in the history of numbers, although Hindu literature suggests it was in use before the time of Christ.

* **Base 16**

Base 16 (hexadecimal) is commonly used as a kind of shorthand to make the usually very long binary numbers of computers more manageable for humans.

* Digits used are the Indian/Arabic numbers 0 thru 9 and the English letters A thru F. A represents 10, B represents 11, C - 12, D - 13, E - 14 and F - 15. Each number occupies a place value. When F is reached, the value goes to 0 and 1 is added to the next place value.
* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F,20, etc.
* Each place value to the left is equal to 16 times the place value to the right which implies that each place value to the right is equal to the place value to the left divided by 16.
* Continuing infinitely <- 65536, 4096, 256, 16, 1
* Hexadecimal is used to represent binary data. Its use applies primarily to the representation of letters and numbers as they are stored in computers. One hex digit represents 4 binary digits. Thus using hex it is possible to represent groups of binary digits (bits) using the numbers 0 thru 9.and the letters A thru F. Octal also represents binary data, but octal represents 3 binary digits. It uses the numbers 0 thru 7 with no letters needed.

|  |
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| **Session 2: Development of the Base-ten Number System** |

After completing this session, you will be able to**: (SO 6) critically analyse the development of the base-ten number system.**

**In this session we are going to explore the following concepts:**

Place value, role of 0 in our number system, patterned nature of whole numbers, history and contestations:

* The development and significance of zero.
* Let’s explore and understand the place value of numbers
* Let’s explore the patterned nature of whole numbers.
* The decimal number system
* The contestations around it
* How and why we use it

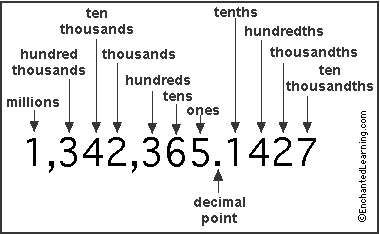
**2.1 The Development and Significance of Zero**

**0 (zero)**, alternatively called naught, nil, nada, ought, zilch, zip, nothing or nought, is both a number and a numeral. It was the last numeral to be created in most numerical systems, as it is not a counting number (which is to say, one begins counting at the number 1) and was in many eras and places represented only by a gap or mark very different from the other numerals. The modern numeral 0 is normally written as a circle or (rounded) rectangle.

0 is the integer that precedes the positive 1, and follows -1. Zero first appeared as a number in Brahmagupta's work dated to 628. Prior to that Babylonians used a space marker that played one of the functions of zero. Babylonians did not have a special symbol for zero. In most (if not all) numerical systems, 0 was identified before the idea of 'negative integers' was accepted.

By the mid-2nd millennium BC, the Babylonians had a sophisticated sexagesimal positional numeral system. The lack of a positional value (or zero) was indicated by a space between sexagesimal numerals. By 300 BC a punctuation symbol (two slanted wedges) was co-opted as a placeholder in the same Babylonian system.

**2.2 The Place Value of Numbers**



**2.3 The Patterned Nature of Whole Numbers**

* **Counting by Two’s**

**You can count by two’s by either:**

* Adding 2 to the previous number or
* Counting and skipping every other number.

The numbers that you would count if you started with 0 and counted by twos would be: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 and so on. Notice that all of the counts are even numbers. The numbers that you would count if you started with 1 and counted by two’s would be: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31 and so on. Notice that all of the counts are odd numbers.

* **Counting by Fives**

Counting by Fives with numbers 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80,

85, 90, 95, 100

**Counting by Fives with words:**

Five, ten, fifteen, twenty, twenty-five, thirty, thirty-five, forty, forty-five, fifty, fifty-five, sixty, sixty-five, seventy, seventy-five, eighty, eighty-five, ninety, ninety-five, one hundred

A number pattern exists when counting by fives. The numbers first end with the number five and then end with a zero. This pattern is repeated 5, 0, 5, 0, 5, 0, etc.

* **Counting by Tens**

Counting by Tens with numbers: 10, 20, 30, 40, 50, 60, 70, 80, 90

Counting by Tens with words:

Ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, one hundred

Number Patterns when counting by Tens:

When you count by tens the numbers create a pattern. All the numbers end with a zero. The first digits are just like the numbers when you count. This pattern gives the numbers 10, 20, 30, 40, 50, etc.

**Writing Numbers up to a Million**

Using the place value table can help you to write large numbers.

Look at the following numbers:

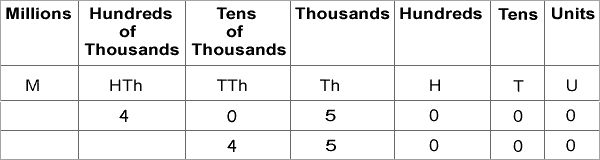
|  |  |
| --- | --- |
| **Numbers in figures** | **Numbers in words** |
| 10 | Ten |
| 100 | Hundred |
| 1 000 | Thousand |
| 10 000 | Ten thousand |
| 100 000 | Hundred thousand |
| 1 000 000 | Million |

You will notice that the numbers are grouped in three figures. There is a space between each group of three figures (counting from right to left). You will sometimes see a comma used to separate the three figures. (If there is no comma in a large number and you have problem saying it, try putting in the comma.)

In South Africa we DO NOT normally use a comma to separate large numbers, we rather use it only to write decimal numbers (we will look at decimals in depth later)

This grouping can help you to say the number 405 000. The first group of three figures is four hundred and five and the last three figures show thousands (since there are three zeros in a thousand). The number is four hundred and five thousand.

Note how important the figure Zero is when we write this number in a place value table:



* **Writing Numbers in Words in Figures**

There are times when you may need to write down a large number in figures that someone has told you in words. Newspaper stories often have large numbers written in figures that may be difficult to make sense of unless you can say them in words.

|  |  |
| --- | --- |
| **Example** | 1. Write five thousand, three hundred and six in figures. Put the 5 in the thousands column and the 3 in the hundreds column. The 6 should go in the units column so make sure you fill the tens column with a 0 to show no tens. The number is 5 306.  2. Write twenty six thousand, seven hundred and fifty in figures. For this number we would start with the tens of thousands column. Any number larger than 9 999 would have 5 figures. Start with the 2 in the tens of thousands column and continue by putting the 6 in the thousands column, the 7 in the hundreds column and the 5 in the tens column. The units’ column must have a 0 to show no units. |

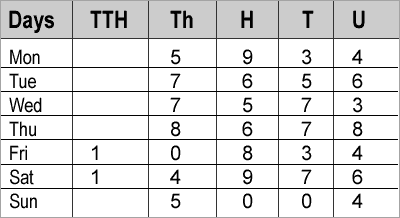
* **Ordering Large Numbers**

When you have a series of large numbers, which are not in number order, it is sometimes difficult to make sense of them. Here is a table showing the daily profits of a supermarket written in order of days

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Day | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
| Profit | R 5 934 | R 7 656 | R 7 573 | R 8 678 | R 10 834 | R 14 976 | R 5 004 |

If these numbers were put into a place value table, it would be easier to arrange them in order.

Look at each column in turn. The figures for Friday and Saturday will be the largest as these have figures in the tens of thousands column. Looking at the thousands column shows that since there is a 4 in the thousand column for Saturday and a 0 in the thousands column for Friday that Saturday has the largest number. Carry on for each of the other numbers.



**Writing Figures in Words**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 0 | Zero | 10 | Ten | 20 | Twenty |
| 1 | One | 11 | Eleven | 30 | Twenty |
| 2 | Two | 12 | Twelve | 40 | Forty |
| 3 | Three | 13 | Thirteen | 50 | Fifty |
| 4 | Four | 14 | Fourteen | 60 | Sixty |
| 5 | Five | 15 | Fifteen | 70 | Seventy |
| 6 | Six | 16 | Sixteen | 80 | Eighty |
| 7 | Seven | 17 | Seventeen | 90 | Ninety |
| 8 | Eight | 18 | Eighteen | 100 | Hundred |
| 9 | Nine | 19 | Nineteen | 1 000  1 000 000 | Thousand  Million |

**2.4 Big Numbers Glossary**

Here are some of the words you may come across to do with big numbers.

* **Place Value**

A figure has a different value when used in different places. For example, in these three numbers, the 4 stands for a different value:

45 The number 4 has a value of 40 (4 tens)

405 The number 4 has a value of 400 (4 hundreds)

54 The number 4 has a value of 4 (4 units)

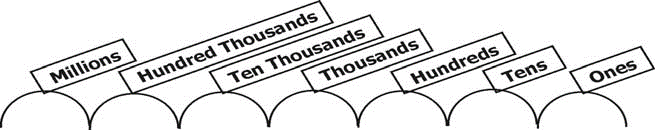
* **Digit**

A figure or a number. 45 is a two-digit number whereas 405 is a three-digit number.

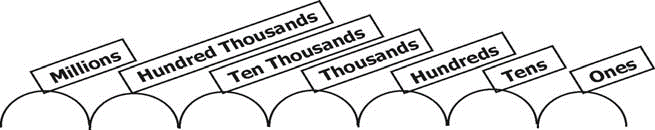
* **Billion**

When we talk about a billion we mean a thousand million or 1 000 000 000. If you see a billion in a news story it is referring to a thousand million. Such big numbers can be difficult to imagine.

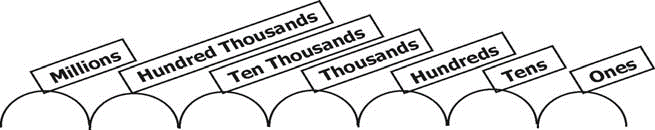
* **Ordering Ascending Numbers**
* Numbers have an order or arrangement. The number two is between one and three. Three or more numbers can be placed in order.
* The order may be ascending (getting larger in value) or descending (becoming smaller in value
* **Seven Digit Numbers**
* How do you write one million rand in numbers? Like this R1 000 000
* The number 1 000 000 has seven digits. These are the seven columns:



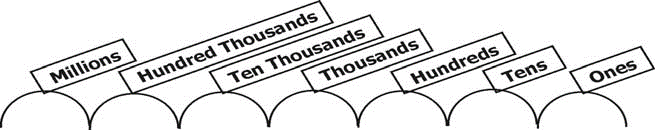
How would you write one million five hundred thousand? That is 1 500 000:



How would you write one million, three hundred and twenty thousand and fifty four? Put place holder zeros into the empty columns like this:



That number is 1 320 054. It would be very different if it was 1 302 054:



That is one million, three hundred and two thousand and fifty four, which is smaller than 1 320 054

1 302 054 < 1 320 054

**2.5 The Decimal Number System**

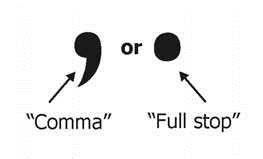
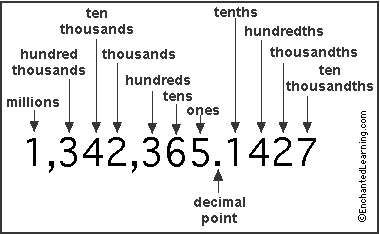
Decimal notation is the writing of numbers in the base-ten numeral system, which uses various symbols (called digits) for ten distinct values (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) to represent numbers. These digits are often used with a decimal separator which indicates the start of a fractional part, and with one of the sign symbols + (plus) or 􀃭 (minus) to indicate sign.

The decimal system is a positional numeral system; it has positions for units, tens, hundreds, etc. The position of each digit conveys the multiplier (a power of ten) to be used with that digit — each position has a value ten times that of the position to its right.

Ten is the number which is the count of fingers and thumbs on both hands (or toes on the feet). In many languages the word digit or its translation is also the anatomical term referring to fingers and toes. In English, decimal (decimals < Lat.) means tenth, decimate means reduce by a tenth, and denary (denarius < Lat.) means the unit of ten.

* **The Contestations Around It**
* The decimal point marks the end of the whole numbers.
* Digits on the left of the point are whole numbers.
* Digits on the right of the point are decimal fractions (fractions of a whole number).
* Where there is no whole number, begin with a zero in the units’ position.
* Digits after the decimal point are usually read individually.
* **How and why we use it**

The decimal separator is a symbol used to mark the boundary between the digit and the fractional parts of a decimal numeral. The decimal separator varies between:

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| **Session 3: Rational & Whole Numbers and Integers** |

After completing this session, you will be able to**: (SO 7 and 8) analyse the relationship between rational and whole numbers, and rational numbers and integers.**

**In this session we are going to explore the following concepts:**

* Different types of numbers.
* The properties of whole numbers.
* The properties of rational numbers.
* The properties of integers.
* The difference between rational and whole numbers.
* The increasing density of each type of numbers.
* Whole numbers as a subset of rational numbers.

**3.1 Different Types of Numbers**

Numbers are, of course, fundamental to mathematics. Even in advanced mathematics where you will often be dealing with the general case rather than a specific case (i.e. using algebra rather than some particular numbers), you often need to specify the type of number that you are using:

* Complex numbers (will learn more later)
* Real numbers (will learn more later)
* Rational numbers:
  + Integers (… -3, -2, -1, 0, 1, 2, 3 …)
  + Whole numbers (0, 1, 2, 3 …)
  + Natural numbers (1, 2, 3, …)
* Irrational numbers (will learn more later)

**3.2 The Properties of Whole Numbers**

Whole numbers are the counting numbers and 0.The whole numbers are 0, 1, 2, 3, 4, 5,

**Place Value**

The position, or place, of a digit in a number written in standard form determines the actual value the digit represents. This table shows the place value for various positions:

|  |  |
| --- | --- |
| Place (underlined) | Name of Position |
| 1 000 | Ones (units) position |
| 1 000 | Tens |
| 1 000 | Hundreds |
| 1 000 | Thousands |
| 1 000 000 | Ten thousands |
| 1 000 000 | Hundred thousands |
| 1 000 000 | Millions |
| 1 000 000 000 | Ten millions |
| 1 000 000 000 | Hundred millions |
| 1 000 000 000 | Billions |

|  |  |
| --- | --- |
| **Example** | The number 721040 has a 7 in the hundred thousand place, a 2 in the ten  thousands place, a one in the thousands place, a 4 in the tens place, and a 0 in both the hundreds and one’s place. |

* **Expanded Form**

The expanded form of a number is the sum of its various place values. For example: 9836 = 9000 + 800 + 30 + 6.

* **Ordering**

Symbols are used to show how the size of one number compares to another. These symbols are < (less than), > (greater than), and = (equals.) For example, since 2 is smaller than 4 and 4 is larger than 2, we can write: 2 < 4, which says the same as 4 > 2 and of course, 4 = 4.

To compare two whole numbers, first put them in standard form. The one with more digits is greater than the other. If they have the same number of digits, compare the most significant digits (the leftmost digit of each number). The one having the larger significant digit is greater than the other. If the most significant digits are the same, compare the next pair of digits from the left. Repeat this until the pair of digits is different. The number with the larger digit is greater than the other.

|  |  |
| --- | --- |
| **Example** | 402 has more digits than 42, so 402 > 42.  402 and 412 have the same number of digits. We compare the leftmost digit of each number: 4 in each case. Moving to the right, we compare the next two numbers: 0 and 1. Since 0 < 1, 402 < 412. |

* **Rounding Whole Numbers**

To round to the nearest ten means to find the closest number having all zeros to the right of the tens place. Note: when the digit 5, 6, 7, 8, or 9 appears in the ones place, round up; when the digit 0, 1, 2, 3, or 4 appears in the ones place, round down.

|  |  |
| --- | --- |
| **Example** | Rounding 119 to the nearest ten gives 120.  Rounding 155 to the nearest ten gives 160.  Rounding 102 to the nearest ten gives 100.  Similarly, to round a number to any place value, we find the number with zeros in all of the places to the right of the place value being rounded to that is closest in value to the original number.  Rounding 180 to the nearest hundred gives 200.  Rounding 150090 to the nearest hundred thousand gives 200000.  Rounding 1234 to the nearest thousand gives 1000.  Rounding is useful in making estimates of sums, differences, etc.  To estimate the sum 119360 + 500 to the nearest thousand, first round each number in the sum, resulting in a new sum of 119000 + 1000. Then add to get the estimate of 120000. |

**3.3 The Properties of Rational Numbers**

Rational numbers consist of every number, which can be written as a ratio of integers. This means that all integers (and hence natural numbers) are also rational numbers, since any whole number can be written as a fraction in the form of itself divided by 1.

**Remark:** The exception to the definition above is if q = 0. In mathematics you cannot divide by 0. Otherwise, you are allowing the concept of infinity into sums and this causes various problems.

We have used the laws of arithmetic to show that if we allow the use of then we will find that 1 = 2 (and hence all numbers are equal!) this is a contradiction, thus we cannot allow ourselves to divide by zero. Infinity does exist, but there is no natural notion of infinity is compatible with the laws of arithmetic. The concept can still be used in some circumstances but you must be very careful with it.

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| **Session 4: Mathematical Symbols & Numerical Models** |

After completing this session, you will be able to**: (SO 1) Express and interpret a range of contexts using mathematical symbols and find applications for numerical models.**

**In this session we are going to explore the following concepts:**

* How we can use mathematical sentences to reflect a situation completely and accurately. We will also look at some everyday problems, and how we can express them with mathematical sentences.
* Explore some numerical models –
  + Equations
  + Expressions
  + Terms.
* How we would apply these numerical models
  + The meaning of symbols
  + Relationships between symbols

**4.1 Mathematical Sentences**

Mathematical Sentences – what are they and why do we use them?

How we can use mathematical sentences to reflect a situation completely and accurately? One of the goals of studying mathematics is to develop the ability to think critically. The study of critical thinking, or reasoning, is called LOGIC. All reasoning is based on the ways we put sentences together. Let's start our examination of logic by defining what types of sentences we will be using.

|  |  |
| --- | --- |
| **Definition** | A **mathematical sentence** is one in which a fact or complete idea is expressed. Because a mathematical sentence states a fact, many of them can be judged to be true or false. Questions and phrases are not mathematical sentences since they cannot be judged to be true or false. **For example:**   * "An isosceles triangle has two congruent sides." is a true mathematical sentence. * "10 + 4 = 15" is a false mathematical sentence. * "Did you get that one right?" is NOT a mathematical sentence - it is a question. * "All triangles" is NOT a mathematical sentence - it is a phrase. |

**There are two types of mathematical sentences:**

* An open sentence is a sentence which contains a variable.
  + "x + 2 = 8" is an open sentence -- the variable is "x."
  + "It is my favourite colour." is an open sentence-- the variable is "It."
* A closed sentence, or statement, is a mathematical sentence which can be judged to be true or false. A closed sentence, or statement, has no variables.
  + "Garfield is a cartoon character." is a true closed sentence, or statement.
  + "A pentagon has exactly 4 sides." is a false closed sentence, or statement.
* A compound sentence is formed when two or more thoughts are connected in one sentence.
  + "Today is a vacation day and I sleep late."
  + "You can call me at 10 o'clock or you can call me at 2 o'clock."
  + "If you are going to the beach, then you should take your sunscreen."
* **Outline:**

**Algebra** provides the basics for all higher Mathematics. You will work with numbers and letters (variables) to form sentences (expressions) that you can solve. The best way to learn Mathematics is by practicing it, so each lesson will include exercises using the skills learned

* **A place to begin:**
  + **Letters** in Mathematics are called variables. They can stand for different numbers at different times.
  + **A mathematical sentence** is called an expression. It can include numbers, variables, signs of operation, and symbols of inclusion.
  + **Signs of operation** tell you what to do to the sentence. The four operations are addition, subtraction, multiplication, and division.
  + **Symbols of inclusion** are parentheses ( ) and brackets [ ].
* **An important caution:**

Be very neat in your calculations. Many an algebra problem is missed because the student misread what he or she had written or did not "line up" the column correctly for subtraction or division. Always double check operations. You don't want to miss a problem because you added incorrectly.

* **Let's Get Started:**
  + To "evaluate" an expression means to find its value, or to solve it. The first rule to learn about algebra is "what to do when." The order in which an expression's operations are done can completely change the answer.
  + When evaluating an algebraic expression, first look for the symbols which show the innermost work. That can be expressed by use of parentheses or brackets. If BOTH parentheses and brackets are present, the parentheses are usually the innermost and should be worked first. For example: 24 + [46 - (2 X 11)] = 24 + [46 - 22] = 24 + 24 = 48

**Let’s look at some everyday problems, and how we can express them with mathematical sentences:**

**4.2 Numerical Models**

* **Equations**

A mathematical sentence with an equals sign to indicate that two expressions name the same number, e.g. 4 + 2 = 6

* **Expressions**

A formula: a group of symbols that make a mathematical statement, and that is an‘expression’ of value, e.g. a – 1 > b + 2

* **Terms**

Parts of an expression or series separated by + or – signs, or the parts of a sequence separated by commas.

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| **Session 5: Solve Everyday Problems Using Estimation & Calculations** |

After completing this session, you will be able to: **(SO 2) Solve a range of everyday problems using estimation and calculations.**

**In this session we are going to explore the following concepts:**

* Mathematical and number problem solving strategies – what are they and how can we use them?
* How to apply a problem solving strategy correctly according to a correct interpretation of the problem situation.
* Learn to estimate.
* Why it is important to calculate accurately.
* How to ensure that your calculation follow some form of logical reasoning process, which is presented clearly.
* Learn to check solutions.

**5.1 Let’s Learn to Estimate first …**

Estimating is an important part of Mathematics and a very handy tool for everyday life. Get in the habit of estimating amounts of money, lengths of time, distances, and many other physical quantities. Rounding off is a kind of estimating.

* **To round off Decimals:**
* Find the place value you want (the "rounding digit") and look at the digit just to the right of it.
* If that digit is less than 5, do not change the rounding digit but drop all digits to the right of it.
* If that digit is greater than or equal to five, add one to the rounding digit and drop all digits to the right of it.
* **To round off Whole Numbers:**
* Find the place value you want (the "rounding digit") and look to the digit just to the right of it.
* If that digit is less than 5, do not change the "rounding digit" but change all digits to the right of the "rounding digit" to zero.
* If that digit is greater than or equal to 5, add one to the rounding digit and change all digits to the right of the rounding digit to zero.
* Estimating, or being able to guess and come close to a correct answer, is an important part of mathematics and a very handy tool for everyday life. You should get in the habit of estimating amounts of money, lengths of time, distances, and many other physical quantities. Rounding is a kind of estimating.
* To round a number you must first find the rounding digit, or the digit occupying the place value you're rounding to. Then look at the digit to the right of the rounding digit. If it is less than 5, then leave the rounding digit unchanged. If it is more than five, add one to the rounding digit. If it is five, the rule is to always round up (add one to the rounding digit). This rule was created to "break the tie" when you are rounding a number that is exactly between two other numbers. These kinds of rules are called "conventions", and are important so we all get the same answer when doing the same problems.
* If you're dealing with a decimal number, drop all of the digits following the rounding digit.
* If you're dealing with a whole number, all the digits to the right of the rounding digit become zero.

**This sounds a lot more complicated than it really is! It's easiest to learn rounding by studying examples:**

|  |  |
| --- | --- |
| **Examples** | **To round the number 16,745.2583 to the nearest thousandth:**   1. First find the rounding digit. This is the "8". 2. You are trying to get rid of the all the digits to the right of the 8, but you want the result to be as accurate as possible. 3. Now look one digit to the right, at the digit in the ten-thousandths place which is "3". 4. See that 3 is less than 5, so leave the number "8" as is, and drop the digits to the right of 8. This gives 16,745.258.   **To round 14,769.3352 to the nearest hundred:**   1. Find the rounding digit, "7". 2. Look at the digit one place to right, "6". 3. Six is more than 5, so this number needs to be rounded up. Add one the rounding digit and change all the rest of the digits to the right of it to zero. 4. You can remove the decimal part of the number too. 5. The result is 14,800.   **To round 365 to the nearest ten:**   1. Find the rounding digit, "6". 2. Look at the digit to the right of the six, "5". 3. Since 365 is exactly halfway between 360 and 370, the two nearest multiples of ten, we need the rule to decide which way to round. 4. The rule says you round up, so the answer is 370. |

* **Rounding Numbers**

A rounded number has about the same value as the number you start with, but it is less exact. For example, 341 rounded to the nearest hundred is 300. That is because 341 is closer in value to 300 than to 400. When rounding off to the nearest rand, R1.89 becomes R2.00, because R1.89 is closer to R2.00 than to R1.00

* **Rules for Rounding**

Here's the general rule for rounding:

* If the number you are rounding is followed by 5, 6, 7, 8, or 9, round the number up. Example: 38 rounded to the nearest ten is 40
* If the number you are rounding is followed by 0, 1, 2, 3, or 4, round the number down. Example: 33 rounded to the nearest ten is 30
* **What Are You Rounding to?**

When rounding a number, you first need to ask: what are you rounding it to? Numbers can be rounded to the nearest ten, the nearest hundred, the nearest thousand, and so on.

**Consider the number 4,827:**

* 4,827 rounded to the nearest ten is 4,830
* 4,827 rounded to the nearest hundred is 4,800
* 4,827 rounded to the nearest thousand is 5,000

**All the numbers to the right of the place you are rounding to become zeros. Here are some more examples:**

* 34 rounded to the nearest ten is 30
* 6,809 rounded to the nearest hundred is 6,800
* 1,951 rounded to the nearest thousand is 2,000
* **Rounding and Fractions**

Rounding fractions works exactly the same way as rounding whole numbers. The only difference is that instead of rounding to tens, hundreds, thousands, and so on, you round to tenths, hundredths, thousandths, and so on.

* 7.8199 rounded to the nearest tenth is 7.8
* 1.0621 rounded to the nearest hundredth is 1.06
* 3.8792 rounded to the nearest thousandth is 3.879

**Here's a tip**: To **avoid getting confused** in rounding long decimals**, look only at the number in the place you are rounding to and the number that follows it**. For example, to round 5.3824791401 to the nearest hundredth, just look at the number in the hundredths place—8—and the number that follows it—2. Then you can easily round it to 5.38.

* **Rounding and Sums**

Rounding can **make sums** **easy**. For example, at a grocery store you might pick up items with the following prices:

If you wanted to know about how much they would cost, you could add up the prices with a pen and paper, or try to add them in your head. Or you could do it the simple way—you could estimate by rounding off to the nearest rand, like this:

By rounding off, you could easily figure out that you would need about . . . R6.00 to pay for your groceries. This is pretty close to the exact number of . . . . . R5.82. As you can see, in finding a round sum, it is quickest to round the numbers before adding them.

Some statisticians prefer to round 5 to the nearest even number. As a result, about half of the time 5 will be rounded up, and about half of the time it will be rounded down. In this way, 26.5 rounded to the nearest even number would be 26—it would be rounded down. And, 77.5 rounded to the nearest even number would be 78—it would be rounded up.

**5.2 Now let’s talk about Calculation…**

* **Adding Even and Odd Numbers**
* even + even = even ; 4 + 2 = 6
* even + odd = odd ; 4 + 3 = 7
* odd + odd = even ; 5 + 3 = 8
* **Subtracting Even and Odd Numbers**
* even - even = even ; 4 - 2 = 2
* even - odd = odd ; 4 - 3 = 1
* odd - odd = even ; 5 - 3 = 2
* **Multiplying Even and Odd Numbers**
* even x even = even ; 4 x 2 = 8
* even x odd = even ; 4 x 3 = 12
* odd x odd = odd ; 5 x 3 = 15
* **Adding and Subtracting Integers**

Looking at a number line can help you when you need to add or subtract integers. Whether you are adding or subtracting two integers, start by using the number line to find the first number. Put your finger on it. Let's say the first number is 3.

* Then, if you are adding a positive number, move your finger to the right as many places as the value of that number. For example, if you are adding 4, move your finger 4 places to the right.
* If you are adding a negative number, move your finger to the left as many places as the value of that number. For example, if you are adding -4, move your finger 4 places to the left.
* If you are subtracting a positive number, move your finger to the left as many places as the value of that number. For example, if you are subtracting 4, move your finger 4 places to the left.
* If you are subtracting a negative number, move your finger to the right as many places as the value of that number. For example, if you are subtracting -4, move your finger 4 places to the right.

**Here are two rules to remember:**

* Adding a negative number is just like subtracting a positive number.
* Subtracting a negative number is just like adding a positive number. The two negatives cancel out each other.
* **Multiplying and Dividing Integers**
* If you multiply or divide two positive numbers, the result will be positive:
* If you multiply or divide a positive number with a negative number, the result will be negative.
* If you multiply or divide two negative numbers, the result will be positive—the two negatives will cancel out each other.
* **Is it an Integer?**

**Only whole numbers are integers**. Therefore, these numbers **can never be** integers: **Fractions, Decimals, Percent’s and Exponents.**

* **Why it is important to calculate accurately:**
* Remember that performing calculations allows no margin for error as each calculation relies on accuracy in order to achieve the correct answer.
* Always take great care when recording calculations as accuracy is the most important part when calculating figures. If any figures in use are incorrectly displayed everything connected to the start figure will throw the end figures out.

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| **Session 6: Solutions Within Different Contexts** |

**In this session we are going to explore the following concepts:**

* How and why we should check and verify our own solutions.
* How and why we should check and verify the solutions of others.
* Explaining the reasoning process clearly.
* Justifying Solutions in terms of the context.
* Ensuring that our solutions are shown to be consistent with estimations and vice versa.

**6.1 Solving Number Problems**

Solving problems can be tricky - that's why they are called problems! How and why should we check and verify our own solutions? How and why should we check and verify the solutions of others? When you first look at a number problem, it is important that you read the question first and then work out if you need to add, subtract, multiply or divide.

* Explaining the reasoning process clearly.
* Justifying Solutions in terms of the context.

Here is a method you can run through in your head to help solve problems:

**Step 1**: Read the problem - Try to picture the problem in your head.

**Step 2**: Organise the calculation - Is it addition, subtraction, multiplication or division?

**Step 3**: Answer the calculation - Use a mental or written method to work it out.

**Step 4:** Answer the problem - Look at what the question asks for and answer it.

* **Single Stage Problems**

Single stage problems need just one calculation to find the answer:

**Step 1:** Read the problem - Your friend came to see you. She had a 10-minute walk followed by a 20-m**inute bus ride. How many minutes did her journey take?**

**Step 2:** Organise the calculation - There is a 10-minute walk and a 20-minute bus ride. The calculation is an addition: - 10 + 20

**Step 3:** Answer the calculation - 10 + 20 = 30

**Step 4:** Answer the problem - Your friend had a 10-minute walk followed by a 20- minute bus ride. Her journey took 30 minutes.

* **Two Stage Problems**

Two stage problems need two separate calculations to find the answer:

**Step 1:** Your friend bought four ice creams. They cost 60c each. How much change should she have from R3?

**Step 2:** "I'm happiest doing addition so I'll see what I can do. If I double the 60c I'll know the cost for two ice creams. Then I'll double the cost of two to get the cost for four. I'll work out the change by adding on until I get to R3".

**Step 3:** 2 ice creams will cost: 60c + 60c = R1.20 4 ice creams will cost: R1.20 + R1.20 = R2.40 to R3 is 60c (10c change to R2.50, then 50c to R3)

**Alternative methods:**

60 x 4 = 240c then 300c – 240c = 60c or: 60c x 4 = R2.40 then R3 - R2.40 = 60c

Step 4: Your friend bought four ice creams costing 60c each. **She should have 60c change from her R3**

* **Three Stage Problems**

Three stage problems need three separate calculations to find the answer.

**Step 1:** Your friend has R5. She buys a magazine costing R1. Inside the magazine is a coupon which offers 50c off some sun cream. In the shop the sun cream is priced at R3.50. Your friend buys the sun cream getting 50c off the price by using the coupon. How much money should your friend have left from her R5?

**Step 2:** "Mmm... I'm confused by the question. I'll read it through a bit at a time to see if it makes sense. Actually it is OK. I think I can do it in my head – she bought the magazine first and took out the coupon. Then in the shop she found the sun cream priced at R3.50. At the checkout she used the coupon to get 50c off the price of the sun cream. She went out with R5, so I can work out how much she should have left. So I will use addition and subtraction".

**Step 3:** Magazine: R1.00 spent Sun cream: R3.50 – 50c (using coupon for 50c off the price) So, R1 spent on a magazine and R3 spent on sun cream = R4 spent in total **R5 - R4 = R1**

**Step 4: Your friend should have R1 change from her R5**

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| **Session 7: Simple and Complex Numerical Expressions** |

After completing this session, you will be able to: **(SO 4) Perform operations on simple and complex numerical expressions.**

**In this session we are going to explore the following concepts:**

* The conventions governing the order of operations
* The Four basic operations in all combinations.
* How to perform number operations without a calculator.
* How to calculate expressions involving exponents without a calculator.
* How to perform number operations with a calculator.
* Remember to check that your solutions are correct.

**7.1 Operations to be performed with a Calculator**

* **The Conventions Governing the Order of Operations**

**BODMAS** can come to the rescue and give us rules to follow so that we always get the right answer:

According to BODMAS, multiplication should always be done before addition, therefore 17 is actually the correct answer according to BODMAS and will also be the answer which your calculator will give if you type in 2 + 3 x 5 <enter>.

I am assuming that you know what everything in BODMAS means apart from "Order". Order is actually a poor word to use here "Power" would be much better though BODMAS doesn't quite have the same ring to it!

**Order means anything raised to the power of a number.**

You may have heard of Einstein's famous equation E = mc2 here it can be said that c is raised to the power 2, or c has order 2 or c is squared (they all mean the same thing!).

**Here's an example to show how to use all the features of BODMAS:**

|  |
| --- |
| **B**rackets  **O**rder  **D**ivision  **M**ultiplication  **A**ddition  **S**ubtraction |

|  |  |
| --- | --- |
| **Example** | **Explain the answer that a calculator would give to the calculation**  **4 + 70/10 x (1 + 2)2 - 1 according to the BODMAS rules:**  **B**rackets gives **4 + 70/10 x (3)2 - 1**  **O**rder gives **4 + 70/10 x 9 - 1**  **D**ivision gives **4 + 7 x 9 - 1**  **M**ultiplication gives **4 + 63 - 1**  **A**ddition gives **67 - 1**  **S**ubtraction gives **66** |

* **The Part of BODMAS most Often Forgotten**

It is quite common for people to forget that brackets always come first – which means you can calculate the answer of whatever is in the brackets before you attempt to calculate the rest of the problem.

Once you've worked out everything in the brackets, normally these sort of problems become very easy!

The mathematical terminology that you might encounter when you are dealing with the order of operations for complicated calculations is as follows:

* Perform operations within parentheses.
* Multiply and divide, whichever comes first, from left to right.
* Add and subtract, whichever comes first, from left to right.

**7.2 The Basic Operations in all Combinations**

* **Commutative Property of Addition and Multiplication**

Addition and Multiplication are commutative: switching the order of two numbers being added or multiplied does not change the result.

* **Associative Property**

Addition and multiplication are associative: the order that numbers are grouped in addition and multiplication does not affect the result.

* **Distributive Property**

The distributive property of multiplication over addition: multiplication may be distributed over addition.

* **The Zero Property of Addition**

Adding 0 to a number leaves it unchanged. We call 0 the additive identity.

* **The Zero Property of Multiplication**

Multiplying any number by 0 gives 0.

* **The Multiplicative Identity**

We call 1 the multiplicative identity. Multiplying any number by 1 leaves the number unchanged.

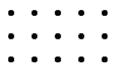
**7.3 Number Operations without a Calculator**

Having defined only one number system we have already arrived at a problem: The difficulty of visualising numbers. Small natural numbers can be recognised as a series of dots, for example:

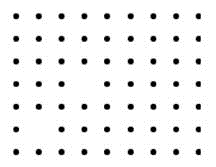
* You should instantly recognise this as five:



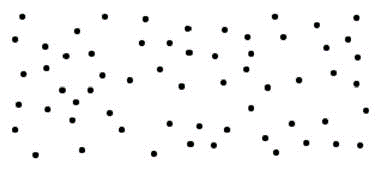
* This you might see as 15 from multiplying 3 by 5:



* However, looking at larger numbers of dots means that you stop recognising them as they are but see them through their properties:



*Looking at this picture you can recognise it as 61 by multiplying 7 by 9 and then subtracting the two holes*



*However, this is much less clear, there is no way that you can quickly see that this as 63, and you have to count every dot.*

* Incredibly large numbers are simply impossible for the human mind to visualise, such as the number of atoms in the universe. To a reasonable degree of accuracy there are about 25000000000000000000000 atoms in the graphite of your pencil.

**7.4 Divisibility - Mathematical Tricks to Learn the Facts**

* **Dividing by 2**
  + All even numbers are divisible by 2. E.g., all numbers ending in 0,2,4,6 or 8.
* **Dividing by 3**
  + Add up all the digits in the number.
  + Find out what the sum is. If the sum is divisible by 3, so is the number
  + For example: 12123 (1+2+1+2+3=9) 9 is divisible by 3, therefore 12123 is too!
* **Dividing by 4**
  + Are the last two digits in your number divisible by 4?
  + If so, the number is too!
  + For example: 358912 ends in 12, which is divisible by 4, thus so is 358912.
* **Dividing by 5**
* Numbers ending in a 5 or a 0 are always divisible by 5.
* **Dividing by 6**
* If the Number is divisible by 2 and 3 it is divisible by 6 also.
* **Dividing by 9**
* Almost the same rule and dividing by 3. Add up all the digits in the number.
* Find out what the sum is. If the sum is divisible by 9, so is the number.
* For example: 43785 (4+3+7+8+5=27) 27 is divisible by 9, therefore 43785 is too!
* **Dividing by 10**
* If the number ends in a 0, it is divisible by 10.

**7.5 How to Calculate Expressions Involving Exponents without a Calculator**

* **Finding Square Roots by Guess & Check Method**

One simple way to find a decimal approximation to, say 􀷭2 is to make an initial guess, square the guess, and depending how close you got, improve your guess. Since this method involves squaring the guess (multiplying the number times itself), it actually uses the definition of square root, and so can be very helpful in teaching the concept of square root.

* **Finding Square Roots using an Algorithm**

There is also an algorithm that resembles the long division algorithm, and was taught in schools in days before calculators. See the example below to learn it. While learning this algorithm may not be necessary in today's world with calculators, working out some examples is good exercise in basic operations for learners, and studying the logic behind it can be a good thinking exercise for most people.

**7.6 How to Perform Number Operations with a Calculator**

A calculator is a device for performing numerical calculations. The type is considered distinct from both a calculating machine and a computer in that the calculator is a special-purpose device that may not qualify as a Turing machine. Although modern calculators often incorporate a general-purpose computer, the device as a whole is designed for ease of use to perform specific operations, rather than for flexibility. Also, modern calculators are far more portable than other devices called computers.

* **A Basic Calculator**

The complexity of calculators varies with the intended purpose. A simple modern calculator, suitable for everyday activities such as shopping or checking a bill, might consist of the following parts:

* A power source, such as a battery or a solar panel
* A display, usually made from LED lights or liquid crystal (LCD), capable of showing a number of digits (typically 8 or 10)
* Electronic circuitry
* A keypad containing:
  + The ten digits, 0 through 9
  + The decimal point
  + The equals sign, to prompt for the answer
  + The four arithmetic functions (namely, addition, subtraction, multiplication and division)
  + A Cancel button, to clear the current calculation
  + On and off buttons
  + Other basic functions, such as square root and percentage (%).